Hiring the Herd: Optimal Unemployment Insurance with Asymmetric Information

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Abstract

This paper characterizes optimal unemployment insurance (UI) in terms of estimable statistics in the presence of negative duration dependence for the unemployed, with endogenous hiring rates generated by asymmetric information. I show how this characterization generalizes the standard Baily (1978) Chetty (2006a) framework. In addition, I construct an approximation for optimal UI as a function of the average elasticity of callback rates with respect to unemployment benefits. There is no estimate of this elasticity in the literature, but I illustrate the responsiveness of optimal UI to this elasticity.

Keywords: unemployment insurance, asymmetric information, negative duration dependence, optimal policy

JEL: H5, J65, D82

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1 Introduction

Social insurance programs in the United States make up 58% of the federal government’s budget, a figure that has grown from under 10% in the 1950s and shows signs of continued rapid growth (Gruber 2009). While insurance against the risk of being too long-lived (Social Security) and against illness (Medicare, Medicaid, and the Children’s Health Insurance Program) are the largest of these programs, unemployment insurance (UI) is important in a labor market where the average number of unemployed Americans in a given month over the past forty years exceeds 8 million. Unemployment insurance is also the social insurance program that has received the most attention from economists interested in connecting theory with data to characterize the optimal design of the program.\(^1\) The standard framework for theoretically characterizing optimal UI with estimable parameters was pioneered by Baily (1978) and generalized by Chetty (2006a). However, these analyses fail to account for endogenous firm behavior, instead assuming that the only choice is that of the unemployed agent with respect to their search effort. In this paper I extend the characterization of optimal UI by introducing an important feedback between the UI program and the hiring behavior of firms.

The firm behavior that I introduce permits “negative duration dependence,” meaning that the longer an individual is out work, the worse are her job market prospects. This can clearly lead to significant inefficiencies in the labor market if employers prefer to hire the newly unemployed over the long-term unemployed, thereby further extending the jobless durations for those with the longest spells. This empirical phenomenon, in the form of callback rates decreasing in unemployment duration, has been carefully documented in a series of recent resume audit studies in Switzerland, the US, and Sweden (Oberholzer-Gee 2008; Kroft et al. 2013; Eriksson and Rooth 2014, respectively). These results are consistent with employer screening models (Vishwanath 1989; Lockwood 1991) where workers are unobservably heterogeneous to employers, who make hiring decisions based on imprecise, but informative private signals about an applicant’s true productivity.\(^2\) I embed such features in a setting with UI to characterize the additional role of policy in a setting with informational asymmetries that generate endogenously determined employment opportunities.

In particular, I assume that finding employment is a two-stage process consisting of effort to apply to a position, followed by the rate at which the employer calls back and offers the employment contract. This differs from standard models of optimal UI because the probability of finding employment depends not only on an unemployed individual’s own effort, but also on the decision of firms to make an offer. The decision

\(^1\) See Chetty and Finkelstein (2013) for an excellent review of this literature.

\(^2\) Kroft et al. (2013) consider other theoretical justifications for negative duration dependence, including human capital depreciation models (Acemoglu 1995; Ljungqvist and Sargent 1998) and ranking models (Blanchard and Diamond 1994; Moscarini 1999). The authors reject these alternative models that fail to predict their empirical finding that duration dependence is stronger in tight labor markets. It is for this reason that I adopt the screening model as more empirically relevant.
by firms to hire an applicant depends on their beliefs about the productivity of that applicant. These beliefs are informed by two statistics: the observable unemployment duration of the applicant and a private signal, which we can interpret as some imperfect measure of quality discerned from the resume. Thus, if high productivity individuals are offered employment at a higher rate (and hence exit unemployment faster) than low productivity workers since they send better private signals, unemployment duration becomes a publicly observable signal of low quality, thereby reducing hiring rates for the longer term unemployed. This rational screening behavior for firms induces negative duration dependence. This behavior is relevant for the design of optimal UI since greater benefits reduce effort, changing the composition of the unemployment pool, and thereby changing hiring rates, which feed back into effort provision, and hence the optimal level of UI.

The main result of this paper is a characterization of optimal UI in terms of estimable statistics in the presence of endogenous hiring rates generated by asymmetric information. I show how the equation for optimal UI generalizes the standard Baily-Chetty framework. In addition, I construct an approximation for optimal UI as a function of the average elasticity of callback rates with respect to unemployment benefits. There is no estimate of this elasticity in the literature, but I illustrate the responsiveness of optimal UI to this elasticity. In future work I intend to empirically estimate this term to complete the implementation of the equation derived in the analysis.

Although this analysis is novel in its consideration of optimal unemployment insurance in the presence of negative duration dependence in the labor market, it is related to a literature that investigates optimal social insurance more broadly in the presence of non-pecuniary externalities. The endogenous callback rate is indeed such an externality, as an individual’s behavior impacts the distribution of types by unemployment duration, which changes callback rates, and hence the return to effort for other agents. Landais, Michaillat and Saez (2010) similarly consider a congestion-type externality, but in their analysis matching frictions in general equilibrium generate an endogenous job-finding rate per unit of search effort, which is dependent on UI. Social multiplier externalities can also impact optimal UI formulas. For example, Lindbeck, Nyberg and Weibull (1999) argue that differences in the size of social insurance systems across developed countries can be partly explained in a model for which individuals’ disutility of work is also a function of the extent to which others benefit from social welfare programs.

The outline for the remained of this paper is as follows. Section 2 presents the model, while Section 3 presents the model, while Section 3
illustrates the dynamic properties of the model via numerical simulation. Section 3 derives an exact equation for optimal UI and Section 5 provides an approximation for optimal UI in terms of estimable parameters. Section 6 concludes.

2 The Model

Consider a discrete-time economy in which an infinitely-lived population of mass 1 is born into unemployment in each period and discounts the future with per-period discount factor \( \beta \). The unemployed agents possess different productive abilities when employed and are described as either high (\( h \)) or low (\( l \)) types, where \( y_i \) is the value of per-period output of a type \( i \in \{ h, l \} \) worker, with \( y_h > y_l > 0 \). These productive abilities are assumed to be private information; each worker knows their own ability, but employers only know the distribution of types within the unemployment population, as in standard adverse selection settings (Akerlof, 1970). Kahn (2013) documents evidence for asymmetric information in the labor market. The share of type \( i \) agents within each new unemployment cohort is denoted by \( \pi_i \in (0, 1) \), with \( \pi_h + \pi_l = 1 \).

There are three possible states for an agent: unemployed, employed, and out of the labor force. Everyone starts unemployed and receives a per-period unemployment benefit of \( b \). In each period of unemployment agents exert effort to obtain employment. Upon successfully being hired, the agent enters the employment state in the subsequent period. Those who do not find employment in each period exit unemployment with exogenous probability \( \delta_u \). This exit can be interpreted as individuals giving up on search and exiting the labor force. Operationally, this assumption generates an expected duration of unemployment benefits that is finite, even with zero effort towards finding a job. Upon employment, a worker receives a per-period wage, \( w \), and pays a per-period tax, \( t \), to finance the unemployment benefits.\(^5\) Note that the wage is assumed to be independent of worker type due to the informational asymmetry. Finally, workers exit employment with exogenous probability \( \delta_e \) at the end of each employment period, due to death or retirement. Importantly, individuals do not re-enter the labor force via unemployment or subsequent employment after leaving their job. As in Lockwood (1991), this assumption helps to reduce the complexity of the model considerably.

It remains to specify the agents’ utility and the process by which they transition from unemployed to employed. Let \( v(\cdot) \) denote the per-period utility of consumption function, where \( v \) is differentiable, strictly increasing, and concave. In addition, let \( k(\cdot) \) be the per-period disutility of unemployment effort function, which is differentiable, strictly increasing, and convex. Moreover, I will normalize utility when out of the

\(^5\) Assume that the UI is not so generous as to induce an individual to reject a job offer and remain in unemployment for perpetuity. I verify that this assumption holds in the numerical simulations of Section 3.
labor force to zero. Denote \( e_i(d) \in [0, 1] \) as the effort choice of an individual of type \( i \in \{h, l\} \) who has been unemployed for \( d \in \mathbb{N}_0 \) periods. There are multiple interpretations for this effort choice. Effort could consist of sending out resumes, networking, research, moving to a different location, etc. What is important and novel in this analysis is that the marginal effectiveness of this effort provision is endogenous to the UI program. In particular, the probability that an agent of type \( i \) with an unemployment duration of \( d \) periods is hired, \( h_i(d) \), is given by

\[
h_i(d) \equiv e_i(d)c_i(d) \quad (1)
\]

where \( c_i(d) \in [0, 1] \) is an endogenous productivity type- and unemployment duration-contingent rate determined by firm screening behavior. There are multiple stages at which firms might screen applicants, but for simplicity, I adopt the following interpretation of the hiring rate. In each period, unemployed agents exert effort, \( e_i(d) \), to find and contact an employer and \( c_i(d) \) is the conditional rate at which individuals are called back to be offered employment. Note that there is no intermediate period between the callback stage and job offer stage - these are the same decision in this model. Thus, two individuals may face different callback rates, but have the same probability of being hired if the agent with fewer callbacks exerts greater effort.

Individuals choose a sequence of effort choices while unemployed to maximize their exponentially discounted expected utility, taking government policy and firm employment offer rates as given. Expected lifetime utility for an individual of type \( i \) at the time of birth into unemployment, \( EU_i \), is therefore given by:

\[
EU_i = v(b) - k(e_i(0)) \\
+ \beta \left( (1 - e_i(0)c_i(0))(1 - \delta_u)(v(b) - k(e_i(1))) + e_i(0)c_i(0)\frac{v(w-t)}{1 - \beta(1 - \delta_e)} \right) \\
+ \beta^2 (1 - e_i(0)c_i(0))(1 - \delta_u) \left( (1 - e_i(1)c_i(1))(1 - \delta_u)(v(b) - k(e_i(2))) + e_i(1)c_i(1)\frac{v(w-t)}{1 - \beta(1 - \delta_e)} \right) \\
+ \cdots \\
= \sum_{\tau=0}^{\infty} \beta^\tau (1 - \delta_u)^\tau \prod_{j=0}^{\tau-1} (1 - e_i(j)c_i(j)) \left[ (v(b) - k(e_i(\tau)) + e_i(\tau)c_i(\tau)\frac{\beta v(w-t)}{1 - \beta(1 - \delta_e)}) \right] \quad (2i)
\]

The first line is utility in the first period of life. The second line is the discounted expected utility in period 2 from not obtaining a job or exiting unemployment in the previous period and again receiving unemployment benefits, and filling a job that yields a constant stream of payoffs \( v(w-t) \). The value of this constant stream of employment payoffs is the sum of the geometric series of per-period payoffs, with common ratio \( \beta(1 - \delta_e) \).
to account for the standard time-preference and the probability that the employment ends. The third line is again the analogous discounted expected utility in the subsequent period, but multiplied by the probability that the individual was still looking for employment in the previous period, \((1 - e_i(0)c_i(0))(1 - \delta_u)\). Gathering the iterative terms and rearranging yields the expression in (2). This expression shows that individuals discount their future stream of expected utility, each period of which is the probability that the agent is unemployed multiplied by the sum of utility in the current period derived from unemployment benefits and the expected utility of finding employment that begins in the subsequent period.

It remains to characterize the role of firms in this economy. To simplify the analysis of optimal UI in this setting, I adopt a partial-equilibrium framework in which the wage, \(w\), paid by firms is exogenous, a standard assumption in the literature pioneered by [Baily, 1978] and [Chetty, 2006a]. In addition, I assume that a firm cannot fire a worker. Thus firms cannot write employment contracts in which wages or employment status are contingent on job performance. Labor is the only cost for a firm, with \(y_l < w < y_h\), implying that a high-type worker is profitable, whereas a low-type worker generates a loss. The firm problem then is restricted to its decision whether or not to offer employment to a job applicant. Upon receiving a job application, the firm observes two pieces of information. First, the firm truthfully observes the duration of unemployment, \(d \in N_0\), of the applicant. Although it is possible for applicants to lie about unemployment duration, I assume that this is verifiable by checking with the government for receipt of benefits or previous employers listed on a resume, for example. Second, the firm receives an imperfect, but informative signal, \(\phi\), about the quality of the potential worker from the application (e.g., quality of cover letter, relevant experience, prestige of education, etc.). Let the distribution of signals depend only on type and be denoted by cumulative distribution functions, \(F_i(\cdot)\) for \(i \in \{h, l\}\), which are known to the employers. The signal is imperfect in the sense that I assume that the intersection of the supports of the distributions is non-empty. Finally, assume that the monotone likelihood ratio (MLR) property holds:

\[
\frac{f_h(\phi)}{f_l(\phi)} \text{ is strictly increasing in } \phi \tag{3}
\]

where \(f_i(\cdot)\) is the probability density function corresponding to \(F_i(\cdot)\) for \(i \in \{h, l\}\). The MLR assumption ensures that the signal is informative, in the sense that the larger the signal, the more likely that the applicant is of high relative to low type.

It is now possible to derive the job offer rule for a firm who receives an application from someone who has been unemployed for \(d\) periods and sends signal \(\phi\). First, denote the share of high types among the
unemployed with duration \(d\) by:

\[
\pi(d) \equiv Pr(y = y_h|d) = \frac{\pi_h \prod_{\tau=0}^{d-1} ((1 - e_h(\tau)c_h(\tau))(1 - \delta_u))}{\sum_{i \in \{h, l\}} \pi_i \left(\prod_{\tau=0}^{d-1} ((1 - e_i(\tau)c_i(\tau))(1 - \delta_u))\right)}
\]

where the first equality is derived by Bayes’ rule and the final expression is a simplification obtained by canceling out the \(1 - \delta_u\) terms. In particular, the numerator is the number of high-type agents who have been unemployed for \(d\) periods, which is simply the number of high-types unemployed for zero periods, \(\pi_h\), multiplied by the probability in each subsequent period up to the previous one, \(d - 1\), that the agent was not hired or left the labor force. The denominator is the total number of unemployed after \(d\) periods, which is the sum of the numerator and the equivalent term for the low types. Note that \(\pi(0) = \pi_h\) and is independent of effort in any period. I assume firms have rational expectations, so that their beliefs about the distribution of types within the unemployment pool is given by \(\pi(d)\) for every \(d\). Then, a firm’s belief about the likelihood that an applicant is of high type given observed unemployment duration, \(d\), and signal, \(\phi\), can be computed by Bayes’ rule as:

\[
Pr(y = y_h|d, \phi) = \frac{f_h(\phi)\pi(d)}{f_h(\phi)\pi(d) + f_l(\phi)(1 - \pi(d))}
\]

A firm therefore calls back and hires an applicant if and only if expected profits are non-negative, or

\[
y_h Pr(y = y_h|d, \phi) + y_l Pr(y = y_l|d, \phi) \geq w
\]

\[
\Leftrightarrow y_h f_h(\phi)\pi(d) + y_l f_l(\phi)(1 - \pi(d)) \geq w(f_h(\phi)\pi(d) + f_l(\phi)(1 - \pi(d)))
\]

\[
\Leftrightarrow (y_h - w)f_h(\phi)\pi(d) \geq (w - y_l)f_l(\phi)(1 - \pi(d))
\]

\[
\Leftrightarrow \frac{f_h(\phi)}{f_l(\phi)} \frac{\pi(d)}{1 - \pi(d)} \geq \frac{w - y_l}{y_h - w}
\]

With the hiring rule established, it is now straightforward to define the job offer probabilities. By the monotone likelihood ratio assumption, a firm’s strategy is also monotone in \(\phi\), conditional on \(d\), so that

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\(\footnote{This formulation implicitly assumes that the firm does not take into account the effort in the present period exerted by the applicant to meet the firm. This is reasonable if we assume that firms have sufficient information to know the distribution of types within the unemployed pool, but not so much information as to know the differential rates at which these types apply for jobs. It is straightforward to adjust this formulation to take such considerations into account, but will leave the results in Proposition[1] and Corollary[1] unchanged.} \)
for \( i \in \{h, l\} \):

\[
c_i(d) = 1 - F_i \left( \frac{f_h(\phi)}{f_l(\phi)} \frac{\pi(d)}{1 - \pi(d)} = \frac{w - y_l}{y_h - w} \right)
\]

(7i)

This function captures a number of features. First, for a given unemployment duration, the callback rate for a high type is greater than for a low type. This follows from the MLR assumption, which implies first order stochastic dominance: \( F_l(\phi) > F_h(\phi) \) for all \( \phi \). In addition, conditional on the length of the unemployment spell, an increase in the relative share of high to low types in that population \( \frac{\pi(d)}{(1 - \pi(d))} \), increases the probability of an offer. Intuitively, with a greater chance of being approached by a high ability type, an employer is more likely to process the private signal as indicating that the worker is indeed high ability and hence more likely to subsequently hire. An increase in the surplus from hiring a high type (or decrease in the loss from hiring a low type) also increases the probability of being hired, as the expected profit increases for any given signal structure or unemployment distribution. It is important to note that the callback probability is defined recursively, as \( c_i(d) \) is defined in terms of \( \pi(d) \), which is in turn defined by the callback rates for shorter unemployment durations, \( c_i(j) \) for \( j = 0, 1, \ldots d - 1 \).

This characterization of firm behavior therefore deviates from the standard optimal framework by endogenizing the marginal benefit of effort. I have abstracted away from more sophisticated behavior on the part of the firm (e.g. setting wage contracts conditional on length of employment, unemployment, observed output, etc.) not to claim that such behavior is unrealistic, but instead to focus on generalizing one important, but restrictive assumption in the standard framework. If unemployment insurance impacts effort choices, which in turn impact the distribution of the unemployment population and hence callback rates that feed back into the effort choice decision, then the design of optimal policy must take such an externality into account.

The government will choose a benefit, \( b \), paid to the unemployed and a tax, \( t \), paid by the employed such that there is budget balance. In particular, it must be that

\[
bU = tN
\]

(8)

where \( U \) and \( N \) are the steady-state size of the unemployed and employed populations, respectively. In steady-state, both \( U \) and \( N \) must be constant. A mass of 1 is born into unemployment in every period and the unemployed exit exogenously at rate \( \delta_u \) or by finding employment. Therefore, in steady-state the
number of unemployed is the sum of individuals of each type at every stage of unemployment duration:

\[
U = \sum_{i \in \{h, l\}} \left[ \pi_i + \pi_i(1 - e_i(0)c_i(0))(1 - \delta_u) + \pi_i(1 - e_i(0)c_i(0))(1 - e_i(1)c_i(1))(1 - \delta_u)^2 + \cdots \right]
\]

\[
= \sum_{i \in \{h, l\}} \left[ \pi_i \sum_{\tau=0}^{\infty} (1 - \delta_u)^\tau \prod_{j=0}^{\tau-1} (1 - e_i(j)c_i(j)) \right]
\]

(9)

To find the number of employed in steady-state, we recognize that \(\delta_e N\) individuals exit employment in each period, but individuals of every unemployment length are entering employment. These two flows must balance in steady-state, so

\[
\delta_e N = \sum_{i \in \{h, l\}} \left[ \pi_i e_i(0)c_i(0) + \pi_i(1 - e_i(0)c_i(0))(1 - \delta_u)e_i(1)c_i(1) \right.
\]

\[
+ \left. \pi_i(1 - e_i(0)c_i(0))(1 - e_i(1)c_i(1))(1 - \delta_u)^2e_i(2)c_i(2) + \cdots \right]
\]

\[
= \sum_{i \in \{h, l\}} \left[ \pi_i \sum_{\tau=0}^{\infty} e_i(\tau)c_i(\tau)(1 - \delta_u)^\tau \prod_{j=0}^{\tau-1} (1 - e_i(j)c_i(j)) \right]
\]

\[
= \sum_{i \in \{h, l\}} \sum_{\tau=0}^{\infty} N_i(\tau)
\]

(10)

where \(N_i(\tau) \equiv \pi_i e_i(\tau)c_i(\tau)(1 - \delta_u)^\tau \prod_{j=0}^{\tau-1} (1 - e_i(j)c_i(j))\) is the number of individuals of type \(i\) entering employment after \(\tau + 1\) periods of unemployment. Therefore,

\[
N = \frac{1}{\delta_e} \sum_{i \in \{h, l\}} \left[ \pi_i \sum_{\tau=0}^{\infty} e_i(\tau)c_i(\tau)(1 - \delta_u)^\tau \prod_{j=0}^{\tau-1} (1 - e_i(j)c_i(j)) \right] = \frac{1}{\delta_e} \sum_{i \in \{h, l\}} \sum_{\tau=0}^{\infty} N_i(\tau)
\]

(11)

Define the unemployment rate as \(u \equiv \frac{U}{U+N}\). With the model established, I will turn in the next section to characterizing the dynamics of effort and hiring rates via numerical simulations.

3 Characterizing Dynamics

In this section I illustrate the sometimes complex interplay between the parameters of the model and the endogenously determined hiring rates and effort provision. Unfortunately, I cannot provide analytical results, so I instead turn to numerical simulations of the model. I will delay until the next section a characterization of optimal policy and instead focus here on the ways in which endogenously determined callback rates over
the unemployment spell impact search effort and the distribution of types within the unemployment pool.

In order to adapt the infinite-horizon model to a finite-horizon numerical simulation, I assume that individuals stay in unemployment for a finite number of periods. If at the end of this time the individual has not found employment, then she exits the labor force. This substitutes for the assumption of the exogenous exit rate, $\delta_u$, in the model of the previous section. In particular, I take a period to be one month and the maximum time in unemployment is six months, the standard length of unemployment benefits in the US.\footnote{An alternative specification would cut off benefits at 6 months, but allow the individual to persist in the unemployment state seeking employment for some period of time after this, but since I abstract away from all other sources of income and self-insurance in this model, I believe my modeling choice to be the most appropriate.}

A drawback of this approach is that I will not be able to study dynamics for the long-term unemployed, although my illustrative results are robust to extending the length of the unemployment benefit contract, as is often done in periods of adverse labor market shocks.

Since I can reasonably calibrate only some of the structural parameters of the model with empirical estimates, I will explore a wide range of possible parameterizations; in Sections 4 and 5 I show how one can evaluate the welfare impact of a small change in UI via estimable statistics. I assume log utility, $v(x) = \ln(x)$, and a disutility of effort while unemployed function given by $k(e) = \eta((1-e) \ln(1-e) + e) - K$ where $\eta, K > 0$ are constants.\footnote{This disutility of effort function satisfies the following boundary conditions: $k'(0) = 0$ and $k'(1) \to \infty$. In addition, in Section 5 I assume that disutility of average effort in the population, $\bar{\tau}$ is approximately zero, therefore justifying a selection of $K$ such that $k(\bar{\tau}) = 0$. For the simulations, I have set $\bar{\tau} = 0.5$. The simulation results are robust to alternative functional forms for utility.}

I set $\eta = 25$, as it fits the unemployment distribution data well, but explore other possible parameterizations below. The monthly discount factor, $\beta$ is set to 0.997, corresponding to an annual discount rate of 4%. In addition I let $w = 3,645$, the average monthly earnings for all private US employees as of May 2014\footnote{See http://research.stlouisfed.org/fred2/series/CES0500000011} and the unemployment benefit is set to $b = 1,700$ to approximate the 46.6% average replacement rate for UI programs in the US\footnote{See http://workforcesecurity.doleta.gov/unemploy/ui_replacement_rates.asp}. To estimate the exit probability from employment, I use the fact that median job tenure is 5.4 years\footnote{As of January 2012 in the US for all adults age 25+. See http://www.bls.gov/news.release/tenure.t01.htm}, which corresponds, to a monthly exit probability of $\delta_e = 1/(5.4\cdot12) = 0.015$.

It remains to specify both the productivity distribution of the individuals newly born into unemployment each period and the structure of the private signals conveyed by job applications to firms. I assume that each new cohort consists of $\pi h$ high productivity individuals who produce a per-period output of value $y_h = w + g$ where $g > 0$ is the per-period profit gained by hiring such an individual. Conversely, $\pi l$ low productivity individuals enter unemployment each period and produce $y_l = w - \pi h \pi l g$. This structure guarantees that ex-ante, before receiving any private information, expected profit from hiring a newly unemployed individual, $\pi_h y_h + \pi_l y_l - w$, equals zero. I also assume that the distribution of signals sent by high and low types are
independent normal distributions with common standard deviation, $\sigma$, and means, $y_h$ and $y_l$, respectively.

In my preferred baseline specification, I set $\sigma = 500$ and $g = 400$, guaranteeing that the types are sufficiently similar and the signal sufficiently imperfect so that employers cannot obviously discern the type of an applicant.

In a series of figures that follow, I show in the left and center panels of each figure the evolution of the callback rates for and effort supplied by high and low types as a function of unemployment duration. In the right panel I plot the share of the unemployed by length of unemployment spell, centering each data point in its corresponding month-long cell. This line plot is superimposed over a histogram of the average distribution of the unemployed in the US from 2001 to 2008, conditional on unemployment spells of less than six months, so as to maintain consistency with my simulation exercise that rules out longer unemployment durations. This provides a check on the model to ensure that the simulation is generating empirically plausible features.

Figure 1 simulates the economy for a range of productivity type distributions: $\pi_h \in \{1/3, 1/2, 2/3\}$. Observe that the callback rates are all decreasing over the unemployment spell, reflecting the key dynamic of negative duration dependence. This arises because as high types exit the unemployment pool at a faster rate than low types, firms update their beliefs that any applicant with a longer unemployment history is

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12 See [http://www.bls.gov/webapps/legacy/cpsatab12.htm](http://www.bls.gov/webapps/legacy/cpsatab12.htm) for data. The widths of the histogram cells are determined by the categories supplied by the Bureau of Labor Statistics: unemployed for less than 5 weeks (0-1.15 months), 5-14 weeks (1.15-3.23 months), and 15-26 weeks (3.23-6 months).

13 This feature obtains in all of the simulations presented in the paper, but it is also possible to choose parameters for which the callback rate is nonmonotonic over the unemployment spell.

14 An important difference between the simulations of this model and the empirical findings of Kroft et al. (2013) is that they find significantly lower average callback rates in the range of 4%-10%. This discrepancy in magnitude stems from the fact that this model assumes an infinite supply of job openings, for which firms are simply deciding whether or not they believe it is profitable to hire an applicant.
more likely to be of low productivity type. Note also that the callback rates for the high types are about an
order of magnitude greater than those for the low types. The low types are unlikely to send a high signal
to convince a potential employer that they are of the high type and subsequently, they receive many fewer
callbacks. Moreover, for a given unemployment duration, the callback rate for high types is increasing in
\( \pi_h \), while for low types, is in general decreasing in \( \pi_h \). Intuitively, with more high types in the population,
firms are more likely to believe a high signal is from a high type and a low signal is from a low type.

Given these callback rates, the unemployed choose their optimal effort provision in each period of potential
unemployment. The story here is a bit more complex. The low types exert decreasing effort over their
unemployment spell due to their low and diminishing marginal effectiveness of effort. For the high types
effort is increasing over the unemployment spell when the share of high types is large, reflecting the fact that
with high callback rates they can postpone intense effort until UI benefits are about to be cut. Note that
due to the declining callback rate, this increase in effort does not result in a “spike” in the exit rate from
unemployment in the final period, which is consistent with recent evidence from Card, Chetty and Weber
(2007). Effort is high and fairly constant, however, for the high types in economies with a lower \( \pi_h \). This
is a consequence of the lower callback rates in these cases, where increased effort substitutes for the lower
chance of receiving an employment offer. Finally, the distributions of the unemployed by duration all fit the
general convexity of the true distribution, but the case with \( \pi_h = 2/3 \) is the best fit. I therefore adopt this
parameter specification in the simulations that follow.

In the next set of simulations, presented graphically in Figure 2, I consider the effect of changing the
precision of the private signals sent by job applicants, as measured by \( \sigma \), the standard deviation of the signal
distributions. The green plots, corresponding to \( \sigma = 500 \), are identical to the green plots in the previous
figure. With a smaller \( \sigma \), firms are better informed about the productivity type of a job applicant, resulting in
high stable callback rates for the high types and low stable callback rates for the low types. This induces high
types to forgo effort until the fourth month of unemployment, at which point effort increases dramatically.
Low types have low and constant effort until month four, at which point the low callback rates and short
future of benefits leads to a decline in effort. Conversely, a larger \( \sigma \) makes it more difficult for firms to screen

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15 The callback rates for low types do not maintain this ordering for intermediate unemployment durations, as endogenous
effort choices change the distribution of types and hence firm beliefs.

16 Contrary to a previous literature that documented a sharp increase in the exit rate from unemployment during the last
period of unemployment benefit receipt, Card et al. (2007) argue this is largely due to a data classification effect and not due
to a behavioral change resulting in re-employment.

17 To formalize this observation, I construct synthetic data sets for the true empirical distribution and each of the simulated
distributions, assuming that the observations are uniformly distributed within each relevant bin. I then perform Kolmogorov-
Smirnov tests of the null hypothesis for equality of the empirical and simulated distribution functions. I reject the null at the
10% level for the distributions corresponding to \( \pi_h \in \{1/3, 1/2\} \). The (corrected) p-values for the tests are 0.037, 0.081, and
0.873, corresponding to \( \pi_h = 1/3, 1/2, \) and \( 2/3 \), respectively.
applicants, resulting in a smaller difference between callback rates for the high and low types. The impact on effort relative to the base case is for the high types to maintain high effort over the unemployment spell and for the low types to begin unemployment with increased effort, but to reduce that effort more quickly as they continue to fail to find employment. The distribution by unemployment duration fits the data better for larger $\sigma$, suggesting that employers are more likely receive imprecise information from job applications.

Note also that almost identical results to those presented in Figure 2 are obtained by instead changing $g$, with an increase in $\sigma$ equivalent to a reduction in $g$. Intuitively, reducing $g$ reduces the distance between the means of the signal distributions for the productivity types, making the signals less informative, as occurs with an increase in $\sigma$.

I conclude this numerical simulation exercise with Figure 3, which considers the implications of changing $\eta$, where the marginal disutility of effort is given by $k'(e) = \eta \ln(1/(1 - e))$. Again, the green plots here are
the same as in the previous two figures. Effort is in general decreasing in $\eta$, since with a greater disutility of effort, individuals find it optimal to exert less effort. Lower effort in turn results in an unemployment population that maintains a larger share of high type individuals, thereby leading to higher callback rates (which feed back into inducing lower efforts). With a low $\eta$, there is a larger share of the unemployed who are only unemployed for the first month, aligning closely with the empirical distribution, but it overestimates the share who have been unemployed for fifteen to twenty-six weeks.

The above simulations illustrate the dynamics introduced in this model that are absent from the literature on optimal unemployment insurance design. By endogenizing a key feature of the job finding process, this model is better able to capture the feedback effects induced by dynamic screening behavior of the firms. A significant drawback of this approach is that there is little empirical evidence to select the appropriate values for the deep parameters of this model. It is therefore why I turn in the next sections to characterizing the optimal level of unemployment insurance using a sufficient statistics approach.

4 Optimal Unemployment Insurance

By “optimal” government policy I mean one that maximizes a standard utilitarian welfare function, subject to the constraints of the economy. In the context of this model, the welfare function, $W$, can be interpreted as the lifetime expected utility of a newly born individual, before knowing their type, entering the steady-state economy:

$$W = \pi_h E U_h + \pi_t E U_t$$  \hspace{1cm} (12)

The social planner problem consists then of choosing an unemployment insurance contract, $(b, t)$, that maximizes \cite{12} subject to: (i) agents of type $i$ choosing an effort sequence, $\{e_i(d)\}_{d=0}^{\infty}$, that maximizes \cite{2}, (ii) $c_i(d)$ is a function of $\{e_i(\tau)\}_{\tau=0}^{d-1}$ for all $d$ \cite{19} and (iii) the balanced budget constraint, $\{8\}$. To simplify the analysis, I will assume in this section that $\beta = 1$ so there is no discounting and to characterize the optimal UI contract, it will be helpful to first introduce two elasticity concepts.

\footnote{Note that the model outlined in the previous section does not impose a zero-profit condition on firms. Thus, any firm profits or losses in the steady-state equilibrium are assumed to be outside the welfare criterion of the social planner. This assumption is made to keep the framework closely aligned with the standard optimal UI literature where the role of the firm is largely outside the scope of the analysis.}

\footnote{The characterization of the solution to this maximization problem is independent of the particular form taken by the callback rates, $c_i(d)$. The only assumption is that they are endogenously determined by effort choices, as in \cite{77} for instance.}
**Definition 1** (Behavioral Elasticity).

\[
\epsilon_{u,b} \equiv \frac{b}{u} \frac{du}{db} \tag{13}
\]

The behavioral elasticity is the standard measure of the distortionary moral hazard cost of supplying social insurance. By increasing benefits to the unemployed, individuals find it optimal to reduce search effort, thus increasing the unemployment rate. Note that neither the tax rate, \(t\), nor the hiring probabilities are held constant in this elasticity measure. Thus, it is possible to empirically estimate this statistic without knowing the precise mechanism by which effort, tax rates, or firm behavior are adjusting. In a standard framework, this is the only relevant elasticity for the determination of optimal social insurance. However, the introduction of an externality in this setting generates another important response to unemployment benefits.

**Definition 2** (Externality Elasticity).

\[
\epsilon_{c,(\tau),b} \equiv \frac{b}{c_{(\tau)}} \frac{dc_{(\tau)}}{db} \tag{14}
\]

The externality elasticity measures the percentage change in the probability that an individual of type \(i\) and unemployment duration \(\tau\) receives a callback from a job application for a one percent increase in the unemployment benefit. When the marginal benefit of search effort is fixed, as is standard, this elasticity is zero. With callback rates endogenously determined by the composition of the unemployment pool, however, an increase in \(b\) impacts effort, which in turn changes the distribution of unemployment and hence the likelihood with which firms offer employment.

Before turning to the main result, it is useful to define the following conditional continuation expected utility for an agent of type \(i\):

\[
EU_i(\tau) \equiv \sum_{m=\tau}^{\infty} \beta^{m-\tau} \left[ \left( v(b) - k(e_i(m)) + e_i(m)c_i(m) \frac{\beta v(w-t)}{1 - \beta(1 - \delta_c)} \right) (1 - \delta_u)^{m-\tau} \prod_{j=\tau}^{m-1} (1 - e_i(j)c_i(j)) \right]
\tag{15}
\]

That is, \(EU_i(\tau)\) measures an individual’s expected future utility, conditional on being unemployed for the previous \(\tau\) periods, starting in that individual’s \(\tau + 1\) period of life. With the above definitions, I now state the main analytic result characterizing the optimal level of social insurance in this economy.
Proposition 1. The optimal UI policy satisfies the equation:

\[
\frac{\epsilon_{u,b}}{1-u} = \frac{v'(b) - v'(w-t)}{v'(w-t)} + \frac{1}{t} \sum_{i \in \{h,l\}} \sum_{\tau=0}^{\infty} N_i(\tau) \frac{\Delta V_i(\tau) v'(w-t) \epsilon_{c_i(\tau),b}}{\delta_e N}
\]

where \(\Delta V_i(\tau) \equiv v(w-t) - \delta_e (1 - \delta_u) EU_i(\tau + 1)\).

Proof. See Appendix (A) \(\square\)

The expression in Proposition 1 modifies the standard Baily-Chetty result for the characterization of optimal unemployment insurance by introducing a new externality correction term. The first two terms in (16) are standard. The moral hazard cost of providing insurance is captured by the left-hand side term, the behavioral elasticity scaled by the inverse of the employment rate. An increase in unemployment benefits reduces effort and hence increases unemployment durations and subsequently the unemployment rate in steady state. The larger this effect, the greater the marginal cost of UI provision. Note that this distortionary effect is magnified by dividing by \(1-u\). This is necessary to capture the fact that an increase in benefits, by extending unemployment duration, reduces the time the agent spends employed and available to pay the tax to fund UI. With a low unemployment rate, however, this effect is small. The first term on the right-hand side of the equality is a standard measure of the insurance benefit. The gap in marginal utilities in the unemployed and employed states, relative to marginal utility while employed, measures the extent to which an agent would benefit from the consumption smoothing provided by insurance. The larger the difference in marginal utilities across states, the greater the benefit. In the extreme case with risk neutral preferences over consumption, this term is zero, implying an optimal benefit level of zero in the standard model (without externalities).

The new feature in this setting is the externality correction term, the final term in (16), which accounts for the effect of UI benefits on endogenous callback rates. Intuitively, the externality correction term measures the average utility change for unemployed individuals who, by facing a change in the callback rate from firms, enter employment with a different probability for a fixed level of effort. To see this directly, first recall from (10) that \(\delta_e N = \sum_{i \in \{h,l\}} \sum_{\tau=0}^{\infty} N_i(\tau)\). Thus, the correction term consists of a weighted average of \(\frac{\Delta V_i(\tau)}{v'(w-t)\epsilon_{c_i(\tau),b}}\) across all types \(i\) and unemployment durations \(\tau\), with the weights given by the share of each type-unemployment duration combination among the total population of agents entering employment. A 1% change in the UI benefit changes the callback rate for an unemployed agent of type \((i,\tau)\) by \(\epsilon_{c_i(\tau),b}\) percent. In the case of a higher callback rate, the agent increases the chance of beginning employment in the subsequent period, where she would earn a future expected lifetime utility of \(v(w-t)/\delta_e\). The counterfactual
is that she fails to find employment, in which case future expected lifetime utility is \((1 - \delta_u)EU_i(\tau + 1)\), where the \(1 - \delta_u\) term accounts for the possibility that the agent exits unemployment before the next period. Hence, the utility gain from obtaining employment is given by \(\Delta V_i(\tau) \equiv v(w - t)/\delta_c - (1 - \delta_u)EU_i(\tau + 1)\). Multiplying \(\Delta V_i(\tau)\) by \(\delta_c\) translates this utility gain into a discounted average (where the discount factor is \(1 - \delta_c\)), which I denote by \(\Delta \bar{V}_i(\tau)\), and dividing by marginal utility \(v'(w - t)\) normalizes the utility units. Thus, the externality correction term isolates the welfare change from callback rates impacted by UI.

Finally, note that this term is scaled by \(1/t\). This adjustment simply reflects the fact that if individuals enter employment faster due to changes in callback rates, individuals spend less time in unemployment receiving benefits. Since \(t = bu/(1 - u)\) in budget balance, a lower tax rate, holding benefits fixed, corresponds to a lower unemployment rate, which magnifies the externality effect. This adjustment follows an analogous logic for the scaling by \(1/(1 - u)\) of the behavioral elasticity.

There are a number of features of this result to clarify and emphasize. First, the equation in Proposition 1 is expressed in terms of estimable sufficient statistics (see Section 5). As is standard with this approach to characterizing optimal policy (Chetty, 2009), the terms in (16) are endogenous to UI. Thus, one must take care when implementing the formula to compute optimal UI benefits. It is also possible to use available empirical estimates to evaluate whether the left or right hand side of (16) is larger, thereby implying a welfare improvement by enacting a small decrease or increase in the current benefit level, respectively. In addition, although I have specified a specific form of the callback rates in (7i), the above equation does not depend on knowing the precise mechanism by which callback rates respond to changes in unemployment benefits. In fact this equation is robust to any partial equilibrium analysis in which the marginal effectiveness of effort while unemployed is endogenously determined. Moreover, the additive form of the externality correction is consistent with the “additivity principle” in which standard formulas for optimal policy include a new additive corrective term in the presence of externalities.\(^{20}\)

The externality correction term takes the same sign as the externality elasticity.\(^{21}\) A positive (negative) externality elasticity implies a larger (smaller) optimal UI benefit. Intuitively, if UI not only provides a consumption smoothing benefit, but also raises callback rates, moving individuals faster into the employed population, then a more generous UI benefit is justified. The converse is true if UI reduces callback rates on average. However, it is not possible to sign the externality elasticity without more structure on the problem.

\(^{20}\)For example, Kopczuk (2003) shows this in a general optimal taxation framework, Landais et al. (2010) similarly consider optimal UI in the context of matching frictions in the labor market, and Farhi and Werning (2013) consider optimal macroeconomic policy in a setting with price rigidities.

\(^{21}\)I am assuming that the optimal UI benefits are not too generous so as to maintain the plausible assumption that \(\Delta \bar{V}_i(\tau) \geq 0\) for all \(i\) and \(\tau\).
An increase in benefits will reduce effort by the unemployed, but the subsequent impact on the distribution of types and tenure among the unemployed could plausibly lead to increased callbacks on average if, for example, the high types respond by reducing effort more than low types, increasing their share among the unemployed and hence callback rates from firms who are more optimistic about receiving an application from a high type. It is also not difficult to construct the opposite case. In the next section I turn to estimating the optimal UI benefit in the presence of this externality effect.

5 Optimal UI Estimation

In order to estimate the optimal benefit level as characterized in Proposition 1, I will first express the optimality equation as an approximation in terms of empirically estimable parameters. The measure I will use for the generosity of unemployment insurance is the replacement rate, \( r = b/(w-t) \), or the ratio of the benefit level to the net-of-tax wage. Then, I will turn to the literature to obtain empirical estimates. Unfortunately, there is no estimate for the externality elasticity. I therefore explore a range of possibilities for this elasticity and the responsiveness of the optimal UI benefit level to it.

To generate an approximation to the equation in Proposition 1, I will make four assumptions that help to simplify the analysis. (A1) Let \( v(c) = \ln(c) \), implying that agents have a coefficient of relative risk aversion equal to 1. (A2) A reasonable approximation can be obtained by replacing all effort choices, \( e_i(\tau) \), and callback rates, \( c_i(\tau) \), in (16) with their population averages, denoted by \( \bar{e} \) and \( \bar{c} \), respectively. An implication of this assumption is that the conditional continuation expected utility, \( E^{U_i(\tau)} \), is independent of \( i \) and \( \tau \), thereby making \( \Delta V_i(\tau) \) similarly independent of \( i \) and \( \tau \). (A3) The disutility of effort at the population average optimal effort, \( k(\bar{e}) \), is zero. Since effort while employed is not modeled, this assumption implies that the disutility of effort in and out of unemployment is of the same magnitude. (A4) The tax rate is sufficiently small so that \( v(w-t) \approx v(w) \). The UI tax is in fact low in most UI systems where unemployment is also small. Given these four assumptions, the following corollary expresses an approximation for the optimal replacement rate, denoted by \( r^* \).

**Corollary 1.** Given assumptions (A1)-(A4), the optimal replacement rate, \( r^* \), is approximately defined by

\[
\frac{\epsilon_{u,b}}{1-u} \approx \frac{1}{r^*} - 1 + \frac{1-u}{u} \frac{(\delta_u - \delta_e(1 - \delta_e)) \ln(w) - \delta_e(1 - \delta_e) \ln(r^*)}{(\delta_u + \delta_e(1 - \delta_e)) r^*} E_{N_i(\tau)}(\epsilon_{c_i(\tau),b})
\]
where the expectation operator is with respect to the distribution of agents of type $i$ and unemployment duration $\tau$ being hired at a given period in steady-state.

Proof. See Appendix (A)

Corollary [11] provides a means to estimate the optimal replacement rate for UI with seven statistics: $\epsilon_{u,b}$, $u$, $w$, $\delta_u$, $\overline{c}$, $\delta_{\epsilon}$, and $E_{N_i(\tau)}(\epsilon_{c_i(\tau),b})$. Since the last statistic, the average externality elasticity, is unknown, I will describe the first six before turning to a quantitative exploration. The behavioral elasticity, $\epsilon_{u,b}$, is well-researched (see Krueger and Meyer (2002) for a review of the empirical literature). In the classic analysis by Meyer (1990) in the United States, an elasticity of 0.9 is found with few controls for individual characteristics, while an elasticity of 0.6 obtains with additional controls. More recently, Landais (2014) employs a regression kink design to estimate an elasticity closer to 0.3. I therefore adopt an intermediate elasticity of 0.5. I also use an unemployment rate of $u = 6\%$. For the wage and exit rate from employment I use the same values adopted in Section [3]: $w = $3,645 and $\delta_u = 0.015$. To construct an estimate for the population average monthly hiring probability, $\overline{c}$, note that from (9) and (10) the unemployment rate satisfies:

$$u = \frac{U}{U + N} \approx \frac{1}{1 - (1 - \delta_u)(1 - \overline{c})} \Rightarrow \overline{c} \approx \delta_u \frac{1 - u}{u}$$

With $\delta_u = 0.015$ and $u = 0.06$, this gives a calibration of $\overline{c} \approx 0.235$.

The final statistic to estimate is the exogenous exit probability from unemployment, $\delta_u$. This rate impacts the expected unemployment length, for which an approximation can be obtained by again replacing the type-specific hiring rates with the population average monthly hiring probability, $\overline{c}$, as follows:

$$E(\text{unemployment duration}) = \sum_{i \in \{h,l\}} \pi_i \left[ \sum_{t=0}^{\infty} (t + 1)(1 - \delta_u)^t \prod_{j=0}^{t-1} (1 - \epsilon_i(j)c_i(j))(1 - (1 - \delta_u)(1 - \epsilon_i(t)c_i(t))) \right]$$

$$\approx \sum_{t=1}^{\infty} t((1 - \delta_u)(1 - \overline{c}))^{t-1}(1 - \delta_u)(1 - \overline{c}))$$

$$= \frac{1}{1 - (1 - \delta_u)(1 - \overline{c})}$$

The average unemployment duration for US adults was 17.3 weeks, or 3.99 months, between 2001 and 2008, the time range chosen so as to avoid the extreme impact of the Great Recession on the labor market. This yields an approximation for $\delta_u = 0.02$ when $\overline{c} = 0.235$. 19
Before turning to the full analysis of the optimal replacement rate let us first note two features of the standard case, which is obtained when the average externality elasticity is equal to zero. In this case the optimal replacement rate is equal to 65%. This is higher than standard estimates for two reasons. First, following Baily [1978], it is standard in the literature to use the coefficient of relative risk aversion, 
\[
\gamma \equiv -v''(w-t)(w-t)/v'(w-t),
\]
as a sufficient statistic for utility, as opposed to imposing a specific functional form as I have done here. Doing so changes the standard consumption smoothing benefit from \(1/r^* - 1\) to the approximation \(\gamma(1 - r^*)\), thereby generating an optimal replacement rate of 47%, with \(\gamma = 1\) for log utility. This poor approximation was noted by Chetty [2006a], who advocates for also using the coefficient of relative prudence, \(\rho \equiv -v'''(w-t)(w-t)/v''(w-t)\) to take account of important third order effects of the utility function. Doing so changes the insurance benefit approximation to \(\gamma(1 - r^*)(1 + \rho(1 - r^*)/2)\), yielding an optimal replacement rate of 62%, which is clearly a much improved estimate. Second, since my analysis abstracts from self-insurance, my estimate is higher than the simulations conducted by Gruber [1997]. Greater risk aversion, prudence, or a smaller behavioral elasticity would all serve to generate an even larger optimal replacement rate.

The dependence of the UI program on the endogenous callback rate externality is presented graphically in Figure 4, which plots the optimal replacement rate as a function of the average externality elasticity using the expression in (17). The relationship is positive and approximately linear, with an increase in

![Figure 4: Optimal Replacement Rate vs. Average Externality Elasticity](image_url)
the average externality elasticity of 0.01 leading to an increase in $r^*$ of about two percentage points. The positive slope is to be expected, for as discussed earlier, if higher benefits induce an increase in callback rates, the externality is positive and more generous benefits are optimal. The slope is sufficiently steep to suggest that even an externality that is modest in magnitude could have a significant impact on the optimal policy. In fact, the optimal replacement rate can reach as low as 30% and as high as 90% over the range of $E_{N_i(\tau)}(c_{e_i(\tau),b}) \in (-0.14, 0.13)$. This analysis underscores the importance of finding a precise estimate for this externality elasticity, and more generally for taking into account externalities or general equilibrium effects when designing optimal policy.

6 Conclusion

This paper builds a tractable model that embeds the empirical phenomenon of negative duration dependence for the unemployed into an analysis of optimal unemployment insurance. I have generalized the standard Baily-Chetty framework in the presence of this externality with an equation characterizing optimal policy in terms of empirically estimable parameters. The optimal replacement rate for UI may be higher or lower than standard estimates, depending on the sign of the average elasticity of callback rates with respect to benefit levels. A precise estimate of this elasticity is needed, as I show that the optimal replacement rate increases (decreases) by about two percentage points for every 0.01 increase (decrease) in this average externality elasticity.

Although this paper contributes to the sufficient statistics literature, there are still important directions for future work. In particular, I assume throughout that the only degree of freedom for policymakers is the level of benefits. Issues such as the length or path of benefits are ignored in this analysis, but remain important areas for investigation. In addition, a general-equilibrium approach would help to understand the role of policy over the business cycle in the presence of negative duration dependence. By expanding this literature of extending classic models to exhibit features that more closely align with empirical reality, it will be possible for economists to provide improved recommendations regarding the optimality of a range of public policies that consume much of the budgets of developed economies.

\footnote{Kroft et al., 2013 document the responsiveness of negative duration dependence over the business cycle in the US and Landais et al., 2010 characterize optimal UI over the business cycle abstracting from issues of duration dependence.}
A Appendix

Proof of Proposition 1 Step 1: Rewrite Welfare. Begin by observing that with no discounting ($\beta = 1$), welfare in (12) can be rewritten in terms of the steady-state populations of unemployed, $U$, and employed, $N$:

$$W = v(b)U + v(w - t)N - \sum_{i \in \{h, l\}} \left[ \prod_{\tau = 0}^{\infty} \left( k(e_i(\tau))(1 - \delta_u) \prod_{j = 0}^{\tau - 1} (1 - e_i(j)c_i(j)) \right) \right]$$

To find an expression for the first order condition $\frac{dW}{db} = 0$, we use the fact that

$$\frac{dW}{db} = \left. \frac{\partial W}{\partial b} \right|_{\tau} + \sum_{i \in \{h, l\}} \sum_{\tau = 0}^{\infty} \left. \frac{\partial W}{\partial c_i(\tau)} \right|_{b, \tau \setminus \{c_i(\tau)\}} \frac{dc_i(\tau)}{db}$$

where $\tau \equiv \{e_h(\tau), c_l(\tau)\}$. Note that the tax rate, $t$, is never held constant in the above derivatives.

Step 2: Computing $\frac{\partial W}{\partial c_i(\tau)}$. By employing the envelope theorem for the agents’ optimal choices of effort and the government budget constraint which requires that $t = bU/N$ we find that:

$$\left. \frac{\partial W}{\partial b} \right|_{\tau} = v'(b)U - v'(w - t)N \left[ \frac{dt}{db} \right]_{\tau}$$

$$= v'(b)U - v'(w - t)N \left[ \frac{U}{N} + b \frac{du}{db} \right]_{\tau} - U \frac{dN}{db} \right|_{\tau}$$

$$= (v'(b) - v'(w - t))U - v'(w - t) \left[ \frac{U}{N} + b \frac{du}{db} \right]_{\tau} (U + N)^2$$

$$= (v'(b) - v'(w - t))U - v'(w - t) \frac{b}{u} \frac{du}{db} \left[ \frac{(U + N)^2}{N} \right] U$$

$$= (v'(b) - v'(w - t))U - v'(w - t) \epsilon_u b \frac{U}{1 - u}$$

where $\epsilon_u b |_{\tau} = \frac{b}{u} \frac{du}{db} |_{\tau}$ is the elasticity of the unemployment rate with respect to the benefit, $b$, holding all hiring rates constant.

Step 3: Computing $\frac{\partial W}{\partial c_i(\tau)}$. Observe that the partial effect of a hiring rate on the UI tax can be
expressed equivalently in terms of the effect on the unemployment rate as:

\[
\frac{dt}{dc_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}} = b \frac{d \left( \frac{U}{N} \right)}{dc_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}} = \frac{b}{N^2} \left( N \frac{dU}{dc_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}} - U \frac{dN}{dc_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}} \right) = \frac{b(U + N)^2}{N^2} \frac{du}{dc_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}} = \frac{b(U + N)^2}{N^2} \frac{u}{c_i(\tau)} \epsilon_{u,c_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}} = \frac{b(U + N)U}{N^2 c_i(\tau)} \epsilon_{u,c_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}} \quad (20)
\]

where \( \epsilon_{u,c_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}} \) is the elasticity of the unemployment rate with respect to the hiring probability for an agent of type \( i \) who has been unemployed for \( \tau \) periods, holding fixed the benefit, \( b \), and all other hiring rates.

With (20) and the envelope theorem for individual optimization, we can now express the partial effect of an increase in a hiring rate on welfare as follows:

\[
\frac{\partial W}{\partial c_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}} = -v'(w-t)N \frac{dt}{dc_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}} + \pi_i c_i(\tau) \frac{v(w-t)}{\delta_c} (1 - \delta_u)^{\tau-1} \prod_{j=0}^{\tau-1} (1 - e_i(j)c_i(j)) \\
- \pi_i e_i(\tau) \sum_{m=\tau+1}^{\infty} \left( (v(b) - k(e_i(m)) + e_i(m)c_i(m) \frac{v(w-t)}{\delta_c}) (1 - \delta_u)^{m-1} \prod_{j=0}^{m-1} (1 - e_i(j)c_i(j)) \right) \\
= -v'(w-t)b(1 + U/N)U \frac{\epsilon_{u,c_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}}}{c_i(\tau)} + \pi_i e_i(\tau) \frac{v(w-t)}{\delta_c} (1 - \delta_u)^{\tau-1} \prod_{j=0}^{\tau-1} (1 - e_i(j)c_i(j)) \\
- \pi_i e_i(\tau) EU_i(\tau + 1) (1 - \delta_u)^{\tau+1} \prod_{j=0}^{\tau+1} (1 - e_i(j)c_i(j)) \\
= \frac{-v'(w-t)bU}{1 - u} \frac{\epsilon_{u,c_i(\tau)} \bigg|_{b,\tau \{c_i, (\tau)\}}}{c_i(\tau)} + \pi_i e_i(\tau) \left( \frac{v(w-t)}{\delta_c} - (1 - \delta_u)EU_i(\tau + 1) \right) (1 - \delta_u)^{\tau-1} \prod_{j=0}^{\tau-1} (1 - e_i(j)c_i(j)) \quad (21)
\]

where \( EU_i(\tau + 1) \) is defined by (15).

**Step 4:** **Combining Steps 1-3** Combine (18), (19), and (21) to obtain the first order condition of the social
planner’s problem:

\[
0 = \frac{dW}{db} - \frac{\partial W}{\partial \tau} + \sum_{i \in \{h,l\}} \sum_{\tau=0}^{\infty} \frac{\partial W}{\partial c_i(\tau)} \bigg|_{b,\tau \in \{c_i(\tau)\}} \frac{dc_i(\tau)}{db} = (v'(b) - v'(w - t))U - v'(w - t) \epsilon_{u,b} U \frac{U}{1 - u} \\
+ \sum_{i \in \{h,l\}} \sum_{\tau=0}^{\infty} \frac{dc_i(\tau)}{db} \left[ -v'(w - t) \frac{bU}{1 - u} \epsilon_{u,c_i(\tau)} \frac{U}{1 - u} \epsilon_{u,c_i(\tau)} \right] \\
+ \pi_i e_i(\tau) \left( \frac{v(w - t)}{\delta_e} - (1 - \delta_u) E U_i(\tau + 1) \right) (1 - \delta_u)^{\tau - 1} \prod_{j=0}^{\tau - 1} (1 - e_i(j)c_i(j))
\]

Dividing (22) by \(Uv'(w - t)\), making use of (14), and rearranging terms, we obtain:

\[
\frac{\epsilon_{u,b}}{1 - u} + \frac{1}{1 - u} \sum_{i \in \{h,l\}} \sum_{\tau=0}^{\infty} \epsilon_{c_i(\tau),b} \epsilon_{u,c_i(\tau)} \bigg|_{b,\tau \in \{c_i(\tau)\}} = \frac{v'(b) - v'(w - t)}{v'(w - t)} \\
+ \frac{1}{bU} \sum_{i \in \{h,l\}} \pi_i \sum_{\tau=0}^{\infty} \epsilon_{c_i(\tau),b} e_i(\tau) c_i(\tau) \left( \frac{v(w - t)}{\delta_e} - (1 - \delta_u) E U_i(\tau + 1) \right) (1 - \delta_u)^{\tau - 1} \prod_{j=0}^{\tau - 1} (1 - e_i(j)c_i(j))
\]

Substituting \(bU\) with \(tN\) (equal by the government budget constraint) and noting that the left-hand side of (23) can be expressed more compactly as the unconditional behavioral elasticity:

\[
\epsilon_{u,b} = \frac{du}{db} u = \left( \frac{\partial u}{\partial \tau} \right) + \sum_{i \in \{h,l\}} \sum_{\tau=0}^{\infty} \frac{\partial u}{\partial c_i(\tau)} \bigg|_{b,\tau \in \{c_i(\tau)\}} \frac{dc_i(\tau)}{db} c_i(\tau)
\]

it follows that the first order condition in (23) can be expressed as

\[
\frac{\epsilon_{u,b}}{1 - u} = \frac{v'(b) - v'(w - t)}{v'(w - t)} + \frac{1}{tN} \sum_{i \in \{h,l\}} \pi_i \sum_{\tau=0}^{\infty} \epsilon_i(\tau)c_i(\tau)(1 - \delta_u)^{\tau} \prod_{j=0}^{\tau - 1} (1 - e_i(j)c_i(j)) \left( \frac{\Delta V_i(\tau)}{v'(w - t)} \right) e_i(\tau),b
\]

where \(\Delta V_i(\tau) \equiv \frac{v(w - t)}{\delta_e} - (1 - \delta_u) E U_i(\tau + 1)\). The desired expression in (16) follows from (24) by using the fact that \(N_i(\tau) \equiv \pi_i e_i(\tau)c_i(\tau)(1 - \delta_u)^{\tau} \prod_{j=0}^{\tau - 1} (1 - e_i(j)c_i(j))\).
Proof of Corollary \[4\] By assumptions (A2) and (A3), I will approximate $\Delta V_i(\tau) = v(w - t) - \delta_e(1 - \delta_u) EU_i(\tau + 1)$ as follows:

$$EU_i(\tau) \approx \sum_{m=\tau}^{\infty} \left( v(b) + \frac{\tau^e v(w - t)}{\delta_e} \right) (1 - \delta_u)^{m-\tau} \prod_{j=\tau}^{m-1} \left( 1 - \frac{v}{\tau^e} \right)$$

$$= \sum_{m=0}^{\infty} \left( v(b) + \frac{\tau^e v(w - t)}{\delta_e} \right) (1 - \delta_u)^m \left( 1 - \frac{v}{\tau^e} \right)$$

$$= \frac{v(b) + \tau^e v(w - t)}{1 - (1 - \delta_u)(1 - \tau^e)}$$

which implies that

$$\Delta V_i(\tau) = v(w - t) - \delta_e(1 - \delta_u) EU_i(\tau + 1)$$

$$= v(w - t) - \frac{\delta_e(1 - \delta_u)}{1 - (1 - \delta_u)(1 - \tau^e)} \left( v(b) + \frac{\tau^e v(w - t)}{\delta_e} \right)$$

$$= v(w - t) - \frac{\delta_e(1 - \delta_u) v(b) + \tau^e(1 - \delta_u) v(w - t)}{\delta_u + \tau^e(1 - \delta_u)}$$

$$= \frac{\delta_u v(w - t) - \delta_e(1 - \delta_u) v(b)}{\delta_u + \tau^e(1 - \delta_u)}$$  \hspace{1cm} (25)

Since the approximation to $\Delta V_i(\tau)$ is independent of $i$ and $\tau$, I will drop the arguments and write the approximation to (16) as:

$$\frac{\epsilon_{u,b}}{1 - u} \approx \frac{v'(b)}{v'(w - t)} - 1 + \frac{\Delta V}{tv'(w - t)} - \frac{\delta_e N_i(\tau) \epsilon_{c_i(\tau),b}}{v(w - t)}$$

$$= \frac{\epsilon_{u,b}}{v'(w - t)} - 1 + \frac{\delta_u v(w - t) - \delta_e(1 - \delta_u) v(b)}{tv'(w - t)(\delta_u + \tau^e(1 - \delta_u))} E_{N_i(\tau)} \left( \epsilon_{c_i(\tau),b} \right)$$  \hspace{1cm} (26)

where the second line uses (25) and the fact that $\delta_e N = \sum_{i \in \{h,l\}} \sum_{\tau=0}^{\infty} N_i(\tau)$, so the expectation operator, $E_{N_i(\tau)}$, is taken with respect to the distribution of agents of type $i$ and unemployment duration $\tau$ being hired at a given period in steady-state. In addition, replace $b$ in (26) with $r(w - t)$ and replace $t$ with $r(w - t)u/(1 - u)$, which is true by the budget constraint, to obtain:

$$\frac{\epsilon_{u,b}}{1 - u} \approx \frac{v'(r^*(w - t))}{v'(w - t)} - 1 + \frac{1 - u \delta_u v(w - t) - \delta_e(1 - \delta_u) v(r^*(w - t))}{u r^*(w - t)v'(w - t)(\delta_u + \tau^e(1 - \delta_u))} E_{N_i(\tau)} \left( \epsilon_{c_i(\tau),b} \right)$$  \hspace{1cm} (27)
And with functional forum assumption on utility, \( A1 \), \((27)\) becomes:

\[
\frac{\epsilon_{u,b}}{1 - u} \approx 1 - \frac{1 - u}{u} \frac{\left( \delta_u - \delta_e(1 - \delta_u) \right) \ln(w - t) - \delta_e(1 - \delta_u) \ln(r^*)}{r^* \left( \delta_u + \frac{\epsilon c}{\delta c(1 - \delta_u)} \right)} E_{N_i(\tau)} \left( \epsilon_{c_i(\tau),b} \right)
\]

Finally, we obtain the desired result by using assumption \((A4)\) to replace \(\ln(w - t)\) with \(\ln(w)\).

\(\Box\)

References


