

PHYSICS Summer Homework 2016

Sister Dominica, OP



First & Last Name:

Due Date:

Dear Physics Students,

Welcome to Physics—where we get to study how our universe works!! In order to do this, we need to effectively use mathematics in order to describe the phenomena we encounter. Please complete the following review activities to prepare for our study.

*The following pages are intended to be a brief review of several topics you have covered in Chemistry, Algebra, and Geometry. Following each review there are short section of questions to practice these skills. **There will be an assessment of the objectives listed below during the first week of classes.** Tutoring will be assigned as needed to achieve mastery of these objectives.*

I look forward to seeing you in August. Have a wonderful rest of your summer!

Sister Dominica, OP

Objectives of Summer Work

I can:

- 1. Manipulate and solve algebraic expressions;*
- 2. Determine the significant figures of a measurement and apply the rules for significant figures in determining a final answer to a problem;*
- 3. Apply Scientific Notation, metric prefixes, and unit conversions;*
- 4. Determine the Best Fit Curve from a set of data points;*
- 5. Apply the properties of Vector Addition, geometry, and right triangle trigonometry to graphically and numerically solve for components and resultants of vectors, as well as the angle of the vector.*

Properties of Real Numbers

Directions: Memorize the following properties and information, be able to apply them, and answer questions below showing each algebraic step. Use separate paper and insert it immediately after this page. Make sure you are comfortable with all basic algebra skills.

Properties of Real Numbers					
Let a , b , and c be real numbers, variables, or algebraic expressions.					
	Property of Addition	Example		Property of Multiplication	Example
1.	Commutative Property $a + b = b + a$	$2 + 3 = 3 + 2$	2.	Commutative Property $a \cdot b = b \cdot a$	$2 \cdot (3) = 3 \cdot (2)$
3.	Associative Property $a + (b + c) = (a + b) + c$	$2 + (3 + 4)$ $= (2 + 3) + 4$	4.	Associative Property $a \cdot (b \cdot c) = (a \cdot b) \cdot c$	$2 \cdot (3 \cdot 4)$ $= (2 \cdot 3) \cdot 4$
5.	Distributive Property $a \cdot (b + c) = a \cdot b + a \cdot c$	$2 \cdot (3 + 4)$ $= 2 \cdot 3 + 2 \cdot 4$			
6.	Additive Identity $a + 0 = a$	$3 + 0 = 3$	7.	Multiplicative Identity $a \cdot 1 = a$	$3 \cdot 1 = 3$
8.	Additive Inverse $a + (-a) = 0$	$3 + (-3) = 0$	9.	Multiplicative Inverse $a \cdot \left(\frac{1}{a}\right) = 1$ Note: a cannot = 0	$3 \cdot \left(\frac{1}{3}\right) = 1$
			10.	Zero Property $a \cdot 0 = 0$	$5 \cdot 0 = 0$

Order of Operations (a) To simply an algebraic expression, order of operations PEMDAS must be followed: 1st Parentheses and other grouping symbols, 2nd exponents, 3rd multiplication and division, 4th addition and subtraction. (b) To solve an equation for a given variable, the order opposite of PEMDAS is followed to isolate the variable. It is critical that the variable is in the numerator when it is isolated, otherwise you solved for the inverse or reciprocal of the variable, not the variable itself.

You will need to be able to algebraically manipulate various equations in physics. Solve for the variable indicated. Manipulate them algebraically as though they were numbers.

a. $p = mv$, $m =$ _____

b. $x_f = x_o + v_o t + \frac{1}{2}at^2$, $a =$ _____

c. $a_c = \frac{v^2}{r}$, $v =$ _____

Hint: How many answers are there?

d. $v^2 = v_o^2 + 2a(s - s_o)$, $a =$ _____

e. $K = \frac{1}{2}kx^2$, $x =$ _____

f. $T_p = 2\pi\sqrt{\frac{\ell}{g}}$, $g =$ _____

g. $F_g = G\frac{m_1m_2}{r^2}$, $r =$ _____

h. $mgh = \frac{1}{2}mv^2$, $v =$ _____

Combining Fractions

Directions: Read the following, be able to manipulate and simplify regular and complex fractions, and **work out** the examples.

To add or subtract fractions, you must first find the lowest common denominator and express each fraction with it.

Example 1	Example 2 <i>From Alma Robinson's Summer Packet</i>
$\frac{a}{x} + \frac{b}{y} = \frac{a}{x} \frac{y}{y} + \frac{b}{y} \frac{x}{x}$ $= \frac{ay}{xy} + \frac{bx}{yx}$ $= \frac{ay + bx}{xy}$	$\frac{a-b}{ab^2} + \frac{a+b}{a^2b} = \frac{(a-b)a}{(ab^2)a} + \frac{(a+b)b}{(a^2b)b}$ $= \frac{(a-b)a}{a^2b^2} + \frac{(a+b)b}{a^2b^2}$ $= \frac{a^2 - ab}{a^2b^2} + \frac{ab + b^2}{a^2b^2}$ $= \frac{a^2 - ab + ab + b^2}{a^2b^2}$ $= \frac{a^2 + b^2}{a^2b^2}$

To multiply fractions, multiply the numerators and multiply the denominators. **To divide fractions,** multiply by the inverse. See the section on complex fractions for more details.

Multiplying Fractions: $\left(\frac{a}{x}\right)\left(\frac{b}{y}\right) = \frac{ab}{xy}$ Dividing Fractions: $\left(\frac{a}{x}\right) \div \left(\frac{b}{y}\right) = \left(\frac{a}{x}\right)\left(\frac{y}{b}\right) = \frac{ay}{xb}$

Questions: Simplify the following. Show all work. *From Alma Robinson's Summer Packet*

1. $\frac{2}{a} + \frac{3}{a-5}$

2. $\frac{2}{x^2 - 36} - \frac{1}{x^2 + 6x}$

3. $\frac{1}{6x} + \frac{2}{3x} - \frac{3}{4x}$

Significant Figures

Overall Directions: Watch the following Video: <http://www.flippingphysics.com/significant-figures.html> and **Take** notes.

Memorize the following rules and **be able to apply** them.

PART I – Directions: State the number of significant figures for the following measurements.

Significant figures of a measurement – those digits that are known with certainty plus the first digit that is uncertain. *Rule 0: All nonzero digits ARE significant.*

Rule 1: Zeros between other nonzero or significant digits ARE significant.

- 1.) 304 m 2.) 3004 m 3.) 30.04 m 4.) 340 004 m

Rule 2: Zeros in front (left) of nonzero digits are NOT significant.

- 5.) 003 m 6.) 00304 m 7.) 0.304 m 8.) 0.000 030 04 m

Rule 3: Zeros that are at the (right) end of a number AND also to the RIGHT of the decimal ARE significant.

- 9.) 304.0 m 10.) 304.00 m 11.) 30.040 m

Rule 4: Zeros at the (right) END of a number but to the LEFT of a decimal are NOT significant UNLESS followed by a written decimal point (that is how one shows they have been measured).

- 12.) 3400 m 13.) 3 000 000 m 14.) 3040. m

- 15.) 340. m 16.) 3040 m 17.) 30.0 m

When a measurement is written in scientific notation, all of the digits ARE significant.

- 18.) 3.0×10^8 m 19.) 3.0 km 20.) 2.050×10^{-3} m

PART II – Directions: Perform the following calculations and write the answer in significant figures.

Addition or subtraction – the final answer should have the same number of DIGITS to the right of the decimal as the measurement with the smallest number of digits to the right of the decimal.

302.22 m	250.6 s	456.8765 m	2.876 m	47.8765 m
+ <u>4.0 m</u>	+ <u>2.35 s</u>	- <u>1 m</u>	- <u>0.110 m</u>	+ <u>0.2 m</u>

Multiplication or division – the final answer should have the same number of SIGNIFICANT FIGURES as the measurement having the smallest number of significant figures.

8 m	26 s	11 m	2.5 m	36.281 m
x <u>4 m</u>	x <u>4 s</u>	x <u>11 m</u>	x <u>3.21 m</u>	x <u>.02 m</u>

Scientific Notation

Directions: Memorize the following forms and rules and be able to apply them.

Very large and very small numbers can be difficult to communicate when written as regular decimals.

Using powers of ten can make expressing large/small numbers easier. In science and engineering fields, scientific notation (a special form which uses powers of ten) is often used to clearly communicate a number and its significant figures.

Form of Scientific Notation & Computations with Powers of Ten	
<u>Form of Scientific Notation: $A \times 10^n$</u> <ul style="list-style-type: none"> A is any number with <u>one</u> digit to the left of the decimal All digits in A <u>are</u> significant n is an integer, equal to number of places the decimal was moved <ul style="list-style-type: none"> n is positive if the number is greater than 1 n is negative if the number is less than 1 	<u>Adding Numbers</u> $A \times 10^x + B \times 10^x = (A + B) \times 10^x$ <ol style="list-style-type: none"> Rewrite so that all have the same exponent Add the numbers before the $\times 10$'s Bring down the power of ten <ul style="list-style-type: none"> Example: $3.0 \times 10^3 \rightarrow 3.0 \times 10^3$ $+ 2 \times 10^2 \rightarrow + 0.2 \times 10^3$ 3.2×10^3
<u>Multiplying Numbers</u> $(A \times 10^x)(B \times 10^y) = (A \times B) \times 10^{x+y}$ <ol style="list-style-type: none"> Multiply the numbers before the $\times 10$'s Add the exponents Example: $(3.0 \times 10^3) \times (2 \times 10^2) = (3.0 \times 2) \times 10^{3+2} = 6 \times 10^5$	<u>Dividing Numbers</u> $(A \times 10^x) / (B \times 10^y) = (A / B) \times 10^{x-y}$ <ol style="list-style-type: none"> Divide the numbers before the $\times 10$'s Subtract the exponents Example: $(6.0 \times 10^3) / (2 \times 10^2) = (6.0 / 2) \times 10^{3-2} = 3 \times 10^1$

The following are ordinary physics problems. Place the answer in scientific notation when appropriate and simplify the units (Scientific notation is used when it takes less time to write than the ordinary number does. As an example 200 is easier to write than 2.00×10^2 , but 2.00×10^8 is easier to write than 200,000,000).

Do your best to cancel units, and attempt to show the simplified units in the final answer.

a. $T_s = 2\pi \sqrt{\frac{4.5 \times 10^{-2} \text{ kg}}{2.0 \times 10^3 \text{ kg/s}^2}} =$ _____

b. $K = \frac{1}{2} (6.6 \times 10^2 \text{ kg}) (2.11 \times 10^4 \text{ m/s})^2 =$ _____

c. $F = \left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(3.2 \times 10^{-9} \text{ C})(9.6 \times 10^{-9} \text{ C})}{(0.32 \text{ m})^2} =$ _____

d. $e = \frac{1.7 \times 10^3 \text{ J} - 3.3 \times 10^2 \text{ J}}{1.7 \times 10^3 \text{ J}} =$ _____

e. $K_{\text{max}} = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (7.09 \times 10^{14} \text{ s}) - 2.17 \times 10^{-19} \text{ J} =$ _____

Metric Prefixes & Unit Conversions

Directions: Be able to apply the following table.

Prefixes may be used to symbolize powers of ten; in this case, the prefix is combined with units. For example:

$$1,000\text{grams} = 1,000\text{g} = 1 \times 10^3\text{g} = 1\text{kg} = 1\text{kilogram}$$

$$0.001\text{meters} = 0.001\text{m} = 1 \times 10^{-3}\text{m} = 1\text{mm} = 1\text{millimeter}$$

<i>Common Metric Prefixes and Powers of Ten</i>			
Decimal	Power	Prefix	Abbreviation
1,000,000,000	10^9	giga	G
1, 000,000	10^6	mega	M
1,000	10^3	kilo	k
1	10^0	--	--
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p

Directions: (a) Read and **be able to apply** the following information on unit conversions.

(b) **Answer** the following questions **showing all work**. After performing numerical calculations, write the “calculator answer” down. Then write your final answer in scientific notation and with significant figures and box/highlight it.

Physics uses the **KMS** system (**SI**: System Internationale). **KMS** stands for kilogram, meter, second. These are the units of choice of physics. The equations in physics depend on unit agreement. So you must convert to **KMS** in most problems to arrive at the correct answer.

Measurements should be in consistent units before doing calculations. Measurement units can be converted from one to an equivalent using a conversion factor. Multiplying any number by the number 1 does not change its value ($6 \times 1 = 6$). Since the numerator and the denominator of conversion factors are equal, conversion factors are equal to the number 1. For example, since 1 meter = 3.281feet

$$\frac{1\text{m}}{3.281\text{ft}} = 1$$

The reason why it is “valid” to multiply a measurement by a conversion factor is because we are simply multiplying by the number 1. If you measured your height as 6.000 feet, this could be converted to meters:

$$\frac{6.000\text{ft}}{1} \times \frac{1\text{m}}{3.281\text{ft}} = 1.829\text{m}$$

Questions

1. How many seconds are in an hour? A day? A year?
2. A speed of 31.254Gm/hour is how many meters/second?
3. A box has a volume of 4,270,000 cm³. What is its volume in cubic meters? [Hint: Be careful!]

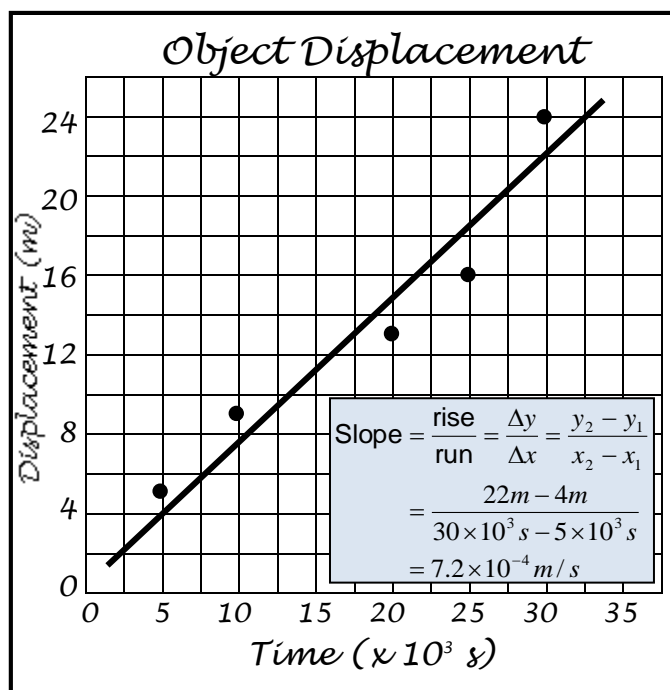
Plotting Data Points & Best Fit Curves

Directions: Read the following information, **complete** the activity and **be able to** plot data points and determine best fit curves.

Graphing Data Once experimental data has been collected, graphical analysis can lead to insights. The steps for graphing data are

1. **T** - **T**itle graph
2. **A** - Label **a**xes with quantity and units (and if necessary factor, such as $\times 10^9$)
 - Independent variable is on the horizontal, x-axis (this is often “time”)
 - Dependent variable is on the vertical, y-axis
3. **S** - Choose and label **s**cale
 - Label all along the axes, either every, or every two, or every five grid lines
4. **P** - Plot points
 - Make them large enough to see
5. **B** - Choose and draw the **b**est fit curve
 - The best fit curve may be linear, but could also be quadratic, inverse, etc.
 - The line should extend through the data range and only slightly beyond
 - If linear, use a ruler
 - NEVER “connect the dots”
 - NEVER draw arrows on the ends of the best fit curve

Example Data Object Displacement	
Time (s)	Displacement (m)
5.0×10^3	5.0
10.0×10^3	9.0
20.0×10^3	12.0
25.0×10^3	16.0
30.0×10^3	24.0



6. **S** – Often you will need to find the slope (see review page on slope)
 - Always use two (convenient) points on the best fit curve; always include units and the factor
 - NEVER use only one point, NEVER use data points unless they fall on the best fit curve

Graphing Activity Part I: Explore curve fitting: [http://phet.colorado.edu/sims/curve-fitting_en.html](http://phet.colorado.edu/sims/curve-fitting/curve-fitting_en.html)
Discuss your findings below.

Part II: Use the information in the data tables to construct graphs according to the above procedure and answer the accompanying questions.

Data Set 1

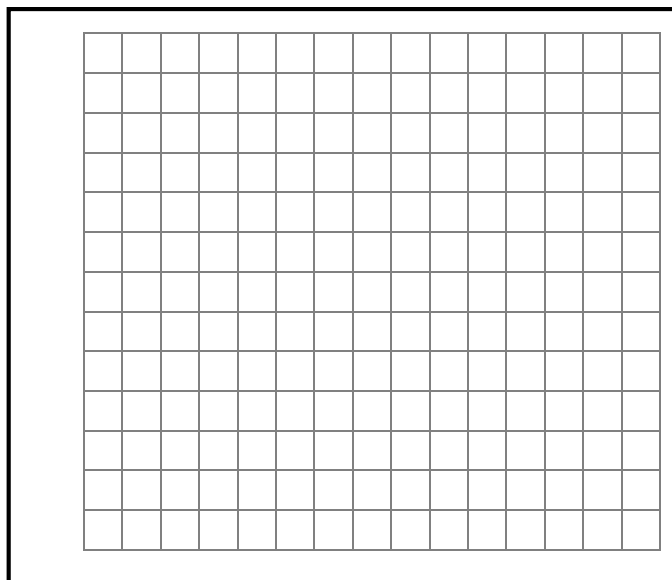
- a. What is the type of best fit curve is used?

Data Set 1	
Time (s)	Speed (m/s)
0.0	0.0
1.0	1.2
2.0	2.7
3.0	3.3
4.0	5.0
5.0	5.6

Data Set 1 continued

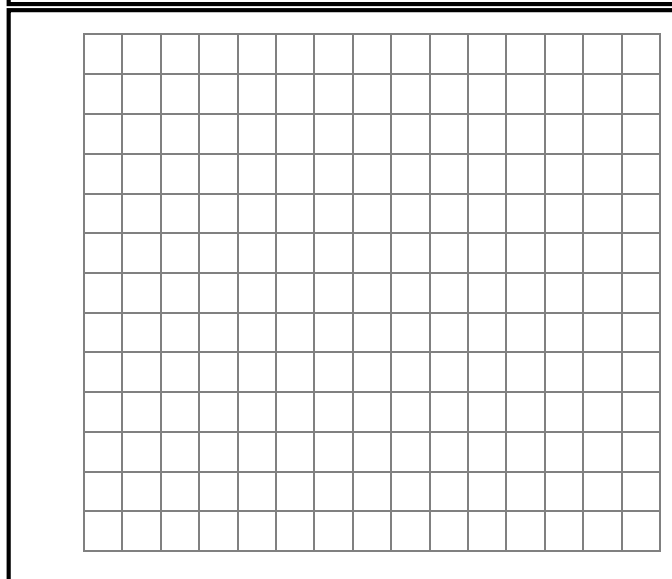
b. Determine the slope of the graph with units. Explain your procedure. Show calculations.

c. The slope of this graph tells us the acceleration of the object being observed since $\text{acceleration} = \text{change in speed} / \text{change in time}$. Describe what the graph would look like if the object were decelerating.

**Data Set 2**

Data Set 2	
Time (s)	Distance (m)
0.0	0
1.0	1.15
2.0	3.6
3.0	9.9
4.0	14.4
5.0	27.5

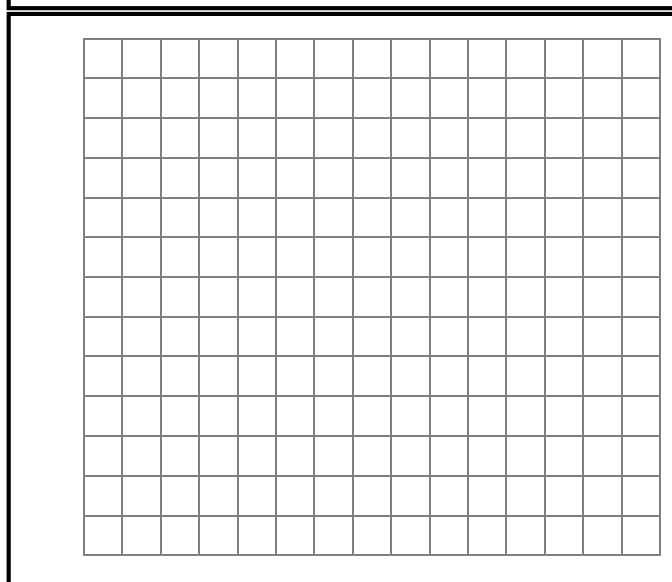
❖ What is the type of best fit curve is used?



Data Set 3 represents data from an experiment in which the velocity of an object in circular motion was kept constant while varying the radius of the motion. The acceleration was then determined.

Data Set 3	
Radius (m)	Acceleration (m/s^2)
0.5	49
1.0	26
1.5	16
2.0	12
2.5	11

❖ What is the type of best fit curve is used?

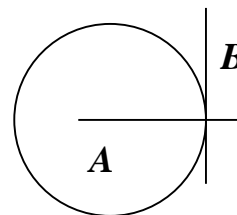


Geometry Practice

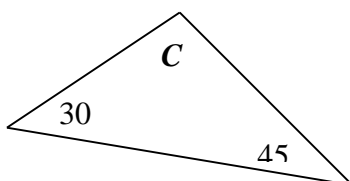
Solve the following geometric problems.

- a. Line **B** touches the circle at a single point. Line **A** extends through the center of the circle.

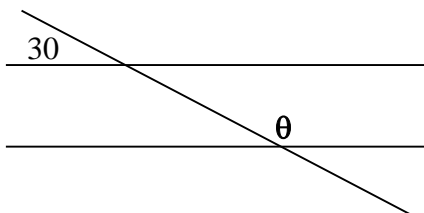
- What type of line is line **B** in reference to the circle? _____
- How large is the angle between lines **A** and **B**? _____



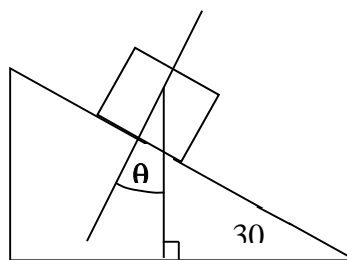
- b. What is angle **C**? _____



- c. What is angle θ ? _____



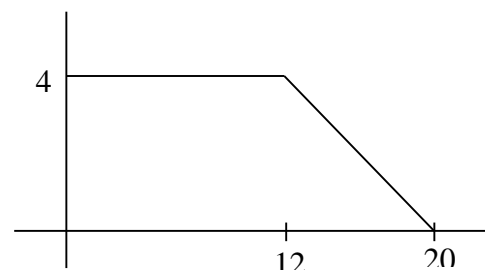
- d. How large is θ ? _____



- e. The radius of a circle is 5.5 cm,

- What is the circumference in meters? _____
- What is its area in square meters? _____

- f. What is the area under the curve (function) at the right?



Using the generic triangle to the right, Right Triangle Trigonometry and the Pythagorean Theorem, solve the following. **Your calculator must be in degree mode.**

- g. $\theta = 55^\circ$ and $c = 32$ m, solve for a and b .

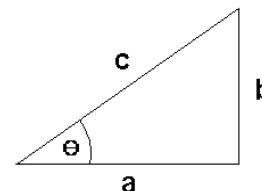
- h. $\theta = 45^\circ$ and $a = 15$ m/s, solve for b and c .

- i. $b = 17.8$ m and $\theta = 65^\circ$, solve for a and c .

- j. $a = 250$ m and $b = 180$ m, solve for θ and c .

- k. $a = 25$ cm and $c = 32$ cm, solve for b and θ .

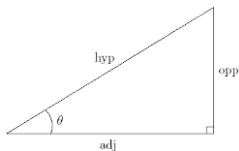
- l. $b = 104$ cm and $c = 65$ cm, solve for a and θ .



Trigonometry

Directions: Read and study the information. Answer the following questions. **Be able to** apply the Pythagorean Theorem, sine, cosine, and tangent functions, and their inverses.

1. There are _____ degrees in a circle. Supplementary angles add to _____ degrees.
2. Complementary angles add to _____ degrees. The sum of the interior angles in a triangle is _____ degrees.
3. A right angle is _____ degrees.
4. A right triangle has one angle which is _____ degrees; the sum of the other two interior angles is _____ degrees.
5. The hypotenuse is the leg of a right triangle which is located across from _____.
Compared to the other sides of a triangle, the hypotenuse is the _____.
6. The Pythagorean Theorem states _____.
7. Draw a Right Triangle. Label the legs of the triangle a & b. Label the Hypotenuse c.
If $a = 3\text{m}$ and $b = 4\text{m}$ how long is c ?
8. Your family is moving to a new house and rents a moving truck. To load the truck ramp was built. If the ramp is 3.3m long and the horizontal distance from the bottom of the ramp to the truck is 2.5m, what is the vertical height of the ramp? Sketch a diagram before solving.
9. Using the labelled triangle below, write down the three equations given for $\sin \theta$, $\cos \theta$, and $\tan \theta$. Here θ represents an angle measured in **degrees**. You will learn the meaning of the trig functions in you math classes if you have not already.



10. You can use your calculator's buttons labeled sin, cos, and tan for these functions and \sin^{-1} , \cos^{-1} , \tan^{-1} buttons for their inverses. Use your calculator to find the answers to these common angles in trigonometry. Keep the answer as a fraction.

a. $\sin 0^\circ = \underline{\hspace{1cm}}$, $\sin 30^\circ = \underline{\hspace{1cm}}$, $\sin 90^\circ = \underline{\hspace{1cm}}$

b. $\cos 90^\circ = \underline{\hspace{1cm}}$, $\cos 60^\circ = \underline{\hspace{1cm}}$, $\cos 0^\circ = \underline{\hspace{1cm}}$

c. $\tan 0^\circ = \underline{\hspace{1cm}}$, $\tan 90^\circ = \underline{\hspace{1cm}}$, $\tan 45^\circ = \underline{\hspace{1cm}}$

Sketch the following triangles and label. Use the trig equations to solve the problems. Show all work!

11. In right triangle XYZ, hypotenuse XY=20.m and angle X=37°. Find the length of leg YZ.
12. In right triangle XYZ, leg YZ=35m and angle Y = 53°. Find the length of the hypotenuse YX.
13. In right triangle XYZ, leg YZ=12m and leg XZ=24m. Find angle X.
14. A ladder 2.7m long leans against a wall and makes an angle of 60° with the ground. Find to the how high up the wall the ladder will reach. Sketch a diagram before solving.

Vectors

Directions: Use your past math learning and the internet to assist you in answering the following.

Be able to apply the concepts.

Italicized portions are from Stephanie Spencer's Summer Packet or Northview HS Summer Packet

There are three types of numerical values used in science and shown in the table.

Type	Scalars		Vectors
Parts	Number Only	Magnitude & Units	Magnitude, Units, & Direction
Example	Coefficient of Friction $\mu = 0.05$	Speed = 5m/s	Velocity = 5 m/s East
Notes	Ratios of values with the same units		

Many of the quantities in physics are vectors. **This makes proficiency in vectors extremely important.**

Magnitude: Size or extent. The numerical value.

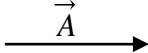
Direction: Alignment or orientation of any position with respect to any other position.

Scalars: A physical quantity described by a single number and units. A quantity described by **magnitude only**.

Examples: time, mass, and temperature

Vector: A physical quantity with **both a magnitude and a direction**. A directional quantity.

Examples: velocity, acceleration, force

Notation: \vec{A} or \overrightarrow{A}  Length of the arrow is proportional to the vectors magnitude.
Direction the arrow points is the direction of the vector.

1. Identify the following as scalar (s) or vector (v).

- | | | |
|------------------|-----------------------------------|-----------------------|
| _____ a) + 3 m/s | _____ c) 50 m/s ² East | _____ e) 51 m/s |
| _____ b) 2 cars | _____ d) -7.01 m | _____ f) 37 m upwards |

Negative Vectors

Negative vectors have the same magnitude as their positive counterpart.

They are just pointing in the opposite direction.



Vector Addition and subtraction

Think of it as vector addition only. The result of adding vectors is called the resultant. \vec{R}

$$\vec{A} + \vec{B} = \vec{R} \quad \overrightarrow{A} + \overrightarrow{B} = \overrightarrow{R}$$

So if A has a magnitude of 3 and B has a magnitude of 2, then R has a magnitude of $3+2=5$.

When you need to subtract one vector from another, think of the one being subtracted as being a negative vector. Then add them.

$$\vec{A} - \vec{B} \text{ is really } \vec{A} + -\vec{B} = \vec{R} \quad \overrightarrow{A} + \overleftarrow{B} = \overrightarrow{R}$$

A negative vector has the same length as its positive counterpart, but its direction is reversed.

So if A has a magnitude of 3 and B has a magnitude of 2, then R has a magnitude of $3+(-2)=1$.

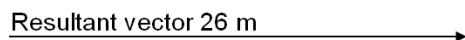
This is very important. In physics a negative number does not always mean a smaller number. Mathematically -2 is smaller than $+2$, but in physics these numbers have the same magnitude (size), they just point in different directions (180° apart).

Adding Collinear Vectors

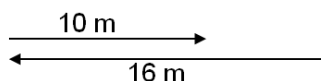
Vectors have both magnitude and direction, thus we must take the direction into account when adding or subtracting vectors. The simplest case occurs when vectors are collinear. Vectors are normally represented by arrows. When adding, we used a tail-to-head relationship. Thus the tail of the vector being added to the original vector is placed at the head of the original vector. If they are in the same direction, simply add the magnitudes of the vectors. The direction will be the same as that of the individual vectors. For example, if someone walked 10 m East and then 16 meters East, we would draw the vectors as:



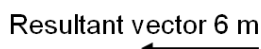
The resultant vector would be 26 m East. It is drawn from the tail of the first vector to the head of the second one.



If vectors are in the opposite direction, add them, keeping in mind they have opposite signs.



The resultant vector would be 6 m West.

**Subtracting Collinear Vectors**

To subtract two vectors, we simply have to take the opposite of the second vector and add it to the first.

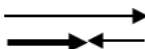
$$30 \text{ m E} - 10 \text{ m E} =$$



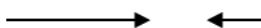
$$30 \text{ m E} + 10 \text{ m W} =$$



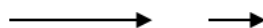
$$20 \text{ m E}$$



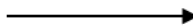
$$30 \text{ m E} - 10 \text{ m W}$$



$$30 \text{ m E} + 10 \text{ m E} =$$



$$40 \text{ m W}$$

**2. Practice:** Add or subtract the vectors as indicated

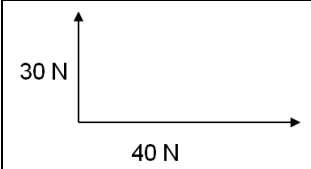
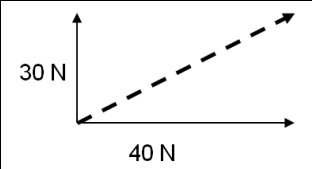
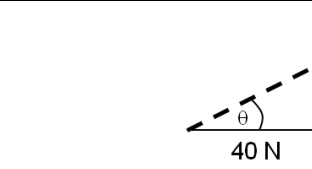
- 15 m/s N + 20 m/s S
- 40 m E + 60 m N
- 28 m N - 15 m S

Adding non-Collinear Vectors

There are two methods of adding vectors that will be discussed further in our 2-dimensional kinematics unit and below.

Adding Perpendicular Vectors

The next simplest case occurs when vectors are perpendicular to each other. We use the Pythagorean theorem to add these vectors. For example, if we had a force of 40 N pushing an object due West and a force 30 N pushing an object due North, we know the object would move along a path between the two forces as shown by the dashed line.

		
A: Perpendicular Vectors	B: Direction of Resultant	C: Graphically add the vectors head-to-tail and draw the resultant vector from the tail of the first to the head of the last.

Arranging the vectors tail-to-head, the resultant is now the hypotenuse, so it would equal 50 N as determined from the Pythagorean Theorem. To find the angle, θ , that the resultant force acts along, we can use the inverse tangent function.

$$\theta = \tan^{-1} (30/40) = 37^\circ \text{ above the horizontal}$$

This tells us that the two original forces could be replaced by a single force of 50 N acting at 37° above the horizontal. The 50 N force at 37° above the horizontal is the sum of the original forces.

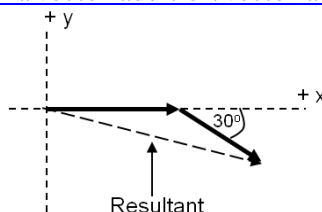
3. Calculate the magnitude and direction of the resultant vectors.

(Hint: draw a picture first then apply the Pythagorean theorem and tangent function; remember vectors have magnitude and direction!)

- 5 m East and 4 m North
- 3 m North and 7 m West
- 5 m/s N + 2 m/s S + 4 m/s E + 6 m/s N

Adding Vectors that are not collinear and not perpendicular

To add vectors such as 70 m due E to 50 m at 30° S of E we can still use the graphical technique of placing the vectors head-to-tail and drawing the resultant vector from the tail of the first vector to the head of the second vector. It does not matter which order we put the original vectors in, the resultant will always be the same (give it a try! http://phet.colorado.edu/sims/vector-addition/vector-addition_en.html).



However, if we want a more accurate numerical answer, we will need to use an algebraic/trigonometric approach. If we break each vector into its x and y components, we can add all of the x-components of the vectors as collinear vectors; similarly with all the y-components. Then we can use the Pythagorean theorem to add the perpendicular vectors that we have calculated. See the next page!

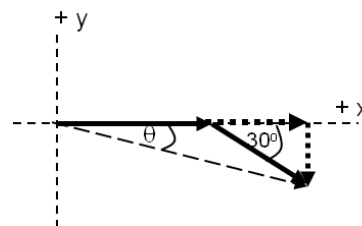
Algebraic/Trigonometric Approach to Finding the Resultant

- A. Start by drawing a picture. Don't skip this step; it will help you avoid direction errors. Set up an x-y system. Label the positive and negative directions.
- B. Use trig to determine the x and y components of each vector. Be sure to take directions into account by using + and - signs.
- The 70 meter vector lies along the +x axis. Its x component is its full length, 70m and it has no y component.
 - The 50 meter vector must be broken into components. Draw the components in the x and y direction to make a right triangle. The original vector is the hypotenuse.
 - The x component of the 50 m vector can be found using the cosine function in this case. The y component can be found using the sine function. Note the negative sign since we are going in the negative y direction.

$$\cos 30 = x/50, \text{ so } x = 50 \cos 30 = 43.3 \text{ m}$$

$$\sin 30 = -y/50, \text{ so } y = -50 \sin 30 = -25 \text{ m}$$

Note: the x component won't always be cosine. It depends on the angle you use.



- C. Make a table to organize your data.
- D. Add the x components and the y components.
- E. Use the Pythagorean Theorem to determine the resultant vector.

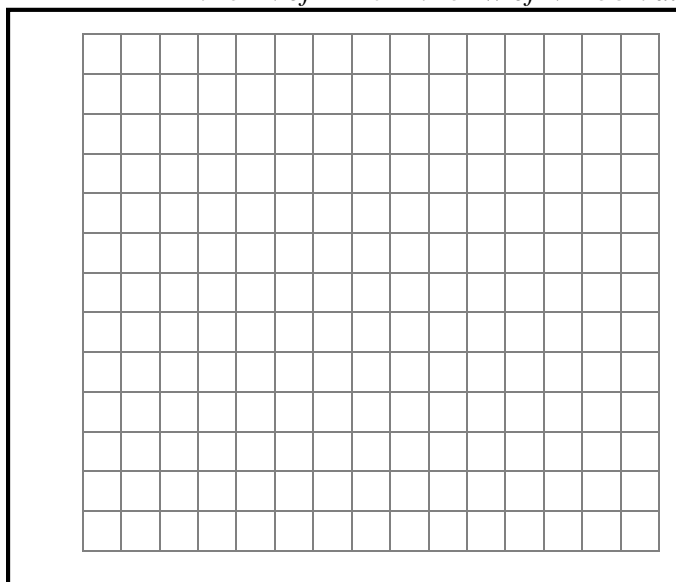
$$\sqrt{113.3^2 + 25^2} = 116 \text{ m}$$

- F. Use the \tan^{-1} function to find the angle
- $$\theta = \tan^{-1} (25/113.3) = 12.4^\circ$$

Vector	x	y
70 m	70	0
50 m	43.3	-25
Resultant	113.3	-25

4. How does one do mathematical (i.e. not graphical) vector addition?
5. Resolve the following vectors into their components. (Hint: draw a picture first then apply the sine and cosine functions.)
- 35 m upwards at an angle of 25° above the horizontal
 - 25m/s at an angle of 300° north of west

6. A cat climbed vertically up a tree trunk 2m and then walked 3m out on a horizontal branch. What is its displacement from the bottom of the trunk? Calculate the magnitude and direction of the resultant vector. (Hint: draw a picture first then apply the Pythagorean theorem and tangent function)
7. Tim climbed 127m upwards at an angle of 62° above the horizontal. (a) What is his horizontal displacement? (b) What is his vertical displacement? (Hint: draw a picture first then apply the sine and cosine functions to resolve the vector into its components.)
8. Add the following vectors both graphically and numerically.
 $22\text{ m } 15^\circ \text{ N of E} + 64\text{ m } 25^\circ \text{ W of N} + 38\text{ m due N}$



THE END!