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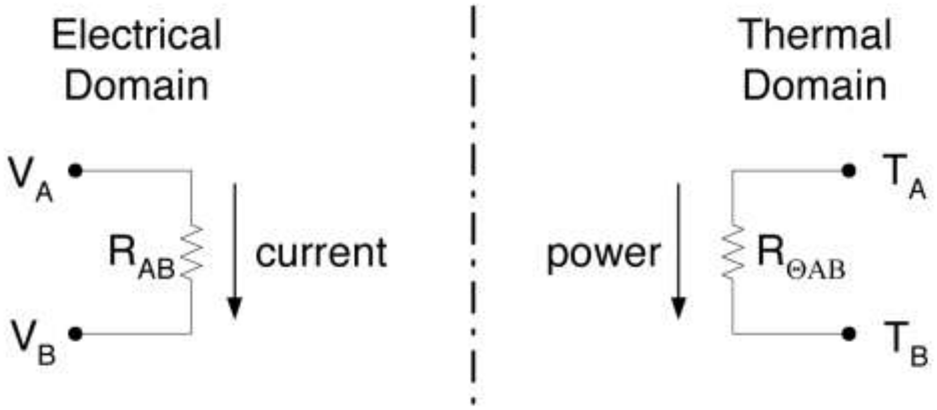
Practical Example of the Equivalent Resistance Method for Thermal Analysis

It is possible to compare heat transfer to current flow in electrical circuits. The heat transfer rate (Q) may be considered as a current flow and the combination of thermal conductivity, thickness of material, and area as a resistance to this flow (R = Δx/kA). The temperature difference (ΔT) is the potential or driving function for the heat flow, resulting in the Fourier equation being written in a form similar to Ohm's Law of Electrical Circuit Theory.

$$\Delta T = Q_{power}\, R_{thermal\, resistance}$$

Figure 1 Electrical / Thermal Domain



The 'resistance term' is what you multiply the power by to get the temperature difference (ΔT). Thermal resistance R has the dimensions of: $\frac{°C}{W}$

Table 1 Equations

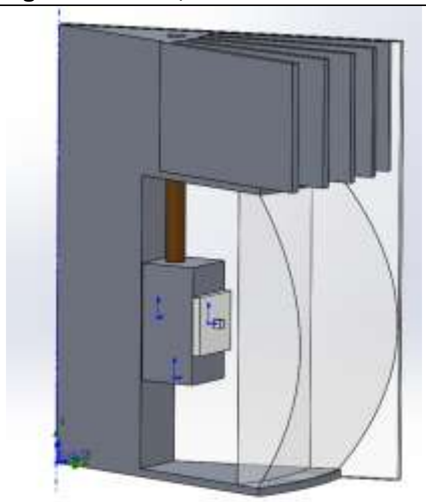
Type of Thermal Transfer	**Equation for Thermal Resistance (K/W)**
Conduction k_{cnd} ≡ conductivity coefficient of the material (W/ K m) Δx ≡ thickness (m) A ≡ area (m^2)	$R_{cnd} = \Delta x/(A\ k_{cnd})$
Convection h_{cnv} ≡ convection coefficient of the surface (W / K m^2)	$R_{cnv} = 1/(A\ h_{cnv})$
	$h_{cnv} = C\left(\frac{\Delta T}{L}\right)^n$ $C_{vertical}$ = 1.42 $C_{horizontal\ up}$ = 1.32 $C_{horizontal\ down}$ = 0.59 n = 0.25 for surface height > 100 mm n = 0.35 for surfaces height < 100 mm n = 0.33 for turbulent flow L for a vertical plate ≡ height $L = \frac{WL}{2(W+L)}$ for a horizontal plate
Net Radiation for a small body in a large area. h_{rad} ≡ radiation coefficient (W / K m^2) ∈ ≡ emissivity (0 to 1) A ≡ 'string' area	$R_{rad} = 1/(\in A\ h_{rad})$ $h_{rad} = 5.67 \text{ x } 10^{-8}\ \frac{W}{m^2\ K^4}\ \frac{(T_1^4 - T_2^4)}{(T_1 - T_2)}$ Temperatures in Kelvin (K) Boltzmann Constant: $5.67 \text{ x } 10^{-8}\ \frac{W}{m^2\ K^4}$
Interface Material (TIM) k_{tim} ≡ interface coefficient or TIM (W / K m^2)	$R_{tim} = \frac{1}{Ak_{tim}}$
Max System Thermal Resistance Q ≡ power in the form of heat (W) $\Delta T = T_{junction\ max} - T_{ambient\ max}$	$R_{system} = \frac{\Delta T}{Q}$
Adding resistance in parallel	$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_n}}$
Adding resistances in series	$R = R_1 + R_2 + R_n$

Example: Using the equivalent resistance method to analyse a high-power LED

Given:
A high power LED light like the one pictured below. The light is powered by 12 high-powered LEDs that are arrange in a circle. To simplify the analysis only one 1/12 of the light will be analysed and thermal symmetry is assumed on the adjacent (1/12) planes. System parameters are:

1. 60°C Ambient temperature
2. 90°C Maximum allowable junction temperature
3. 6 W Power, 50% efficiency

Figure 1 1/12 section



Example picture of light

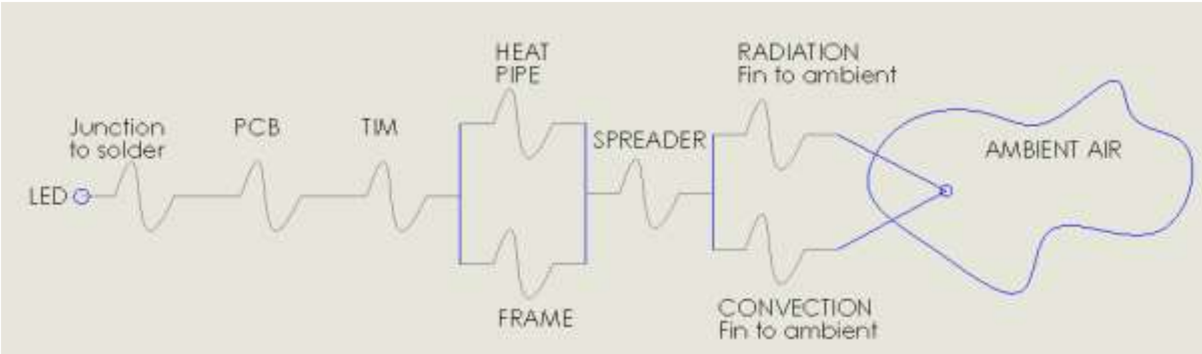
Required:

A. Determine the required thermal system resistance from the junction to the ambient environment.
B. Make first order resistance calculation of the each element.
C. Determine if the design resistance is equal to or less than the required resistance.
D. Refine the design using CFD software.

Solution:

1.1 A resistor diagram is created of the thermal path.

Figure 2



1.2 The required system thermal resistance is calculated below. The mechanical design must result in a system thermal resistance that is equal to or less than the required thermal resistance for the system ($\boldsymbol{R_{required\ system}}$).

$$R_{required\ system} = \frac{\Delta T}{Q_{elect}\ (1 - LED\ efficiency)}$$

$$R_{required\ system} \leq \frac{30°C}{6W\ (1 -\ 0.5)} = 10\ °C/W$$

1.3 Next the resistance is calculated for each element in the thermal path.

1.4 $R_{j\text{-}s}$ junction to solder point

Rj-s this value is obtained from the LED manufacture or testing based on JEDEC standards. The manufactures R_{js} value is determined from a specific test setup that may not match your application. So an understanding of the manufactures test configuration is needed.

1.5 R_{pcb} through the pcb

This value is obtained from the PCB / MPCB manufacture or CDF software that has a PCB calculator.

1.6 R_{tim} thermal interface material / contact resistance

$$\boldsymbol{R_{tim} = \frac{1}{A\,h_{tim}} = \frac{C_r}{A}}$$

The terms 'contact coefficient' h_{tim} (W/m2°C) or a specific 'contact resistance' C_r (m^2 °C/W), or a thermal conductivity and thickness may be specified from the TIM manufacture or engineering references.

1.7 R_{hp} through the heat pipe

The effective thermal resistance value is obtained from the heat pipe manufacture or can be estimated by assuming the heat pipe is a solid. Typical effective thermal conductivity values range between 5,000 to 200,000 W/mK. For this example an effective thermal conductivity of 20,000 W/mK and a diameter of 5 mm and a length of 30 mm will be used. The conduction resistance equation is used.

$$R_{hp} = \Delta x/(A\,k)$$
$$R_{hp} = 0.03\,m/(1.27\,x\,10^{-5}\,m^2\,x\,20{,}000\,W/mK)$$
$$R_{hp} = 0.12\,\frac{°C}{W}$$

1.8 R_{fra} through the aluminium frame with a conductivity of 200 W/mK

$$R_{fra} = \Delta x/(A\,k)$$
$$R_{fra} = .05\,m/(2.4\,x\,10^{-4} m^2\,x\,200\,\frac{W}{mK})$$
$$R_{fra} = 1.1\,\frac{°C}{W}$$

1.9 R_{sp} through the aluminium heat sink spreader plate k = 200 W/mK

$$R_{sp} = \Delta x/(A\,k)$$

1.10 Solving for convection and radiation resistance takes a little skulduggery. Note that the convection coefficient h_{con} and the radiations coefficient h_{rad} are dependant on knowing the surface temperature - which you do not know. What to do?

R_{cnv} fin convection to air (for this example fin efficiency is 80%)

$$R_{cnv} = 1/(A\,h_{cnv}\,eff)$$

$$R_{cnv} = 1/(A\ 1.42\left(\frac{T_2}{.030}\right)^{.35}\ 0.8)$$

R_{rad} fin radiation to air

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$$R_{rad} = 1/(\in A\ h_{rad})$$

$$R_{rad} = 1/(\in A\ 5.67 \text{ x } 10^{-8}\ \frac{W}{m^2\ K^4}\ \frac{(T_1^4 - T_2^4)}{(T_1 - T_2)}$$

Add the parallel resistances for convection and radiation and solve for ΔT (fin to air) as shown below.

$$\Delta T = Q\ R_{rad+cnv} = Q\left(\frac{1}{\frac{1}{R_{rad}} + \frac{1}{R_{cnv}}}\right) = \frac{Q}{\frac{1}{\frac{1}{\in A\ h_{rad}}} + \frac{1}{\frac{1}{A_{fin}\ h_{cnv}\ E_{fin}}}}$$

$$\Delta T = \frac{Q}{\in A\ h_{rad} + A_{fin}\ h_{cnv}\ E_{fin}}$$

$$\boldsymbol{\Delta T} = \frac{Q}{0.8\ x\ A\ x\ 5.67\ \text{x}\ 10^{-8}\ \left(\frac{W}{m^2\ K^4}\right)\frac{(\boldsymbol{T_{fin}^4} -\ 333^4\ )}{(\boldsymbol{T_{fin}} -\ 333\ \ )} + A_{fin}\ x\ 1.42\left(\frac{\boldsymbol{T_{fin}} -\ 333}{.030}\right)^{.35}\ 0.8}$$

The full equation is shown above so that it can be seen that $\boldsymbol{T_{fin}}$ is in ΔT, h_{rad} and h_{cnv} . How do you solve for ΔT when the T_{fin} term is in both the answer and the argument? Use the table below to iterate:

Iteration #1 guess at ΔT then calculate h_{cnv}, R_{cnv}, h_{rad} , R_{rad} and $\Delta T_{(fin\ to\ air)}$
Iteration #2 copy the calculated $\Delta T_{(fin\ to\ air)}$ into the Iteration #2, Guess ΔT cell and re-calculate h_{cnv}, R_{cnv}, h_{rad} , R_{rad} and $\Delta T_{(fin\ to\ air)}$

Iteration #3… Repeat the iteration process until the Guess ΔT value is equal to the calculated $\Delta T_{(fin\ to\ air)}$

Table 2 Determine Rcnv and Rrad

ITERATION	**ΔT**	**Tfin (K)**	**hcnv**	**Rcnv**	**hrad**	**Rrad**	**Calc ΔT**
1	10.00	343	10.85	7.09	8.76	27.98	17.0
2	16.97	350	13.05	5.89	9.04	27.12	14.5
3	14.52	348	12.36	6.22	8.94	27.42	15.2
4	15.22	348	12.56	6.12	8.97	27.33	15.0
5	15.01	348	12.50	6.15	8.96	27.36	15.1
6	15	348	12.52	6.14	8.96	27.35	15

Notes:

1. $T_{fin} = \Delta T + T_{air}$ (K)
2. T_{air} for this example is 333 K
3. Read the radiation and convection resistances (R_{rad}, R_{cnv}) from last iteration and enter in the RESISTANCE TABLE.
4. Radiation from the fin to the ambient is small.

2 ADD UP ALL THE RESISTANCES resulting in the SYSTEM RESISTANCES

Table 3 Resistance Tabulation

Element	**Legs of parallel elements**	**Resistance (°C/W)**
Junction to solder point		2.5
Through PCB / MPCB		4.4
Through TIM		0.01
[1]Through Heap Pipe	0.12	
[1]Through Frame	1.1	
Heap pipe + Frame		0.11
Through Heat Spreader		0.02
Fin radiation to ambient	27	
Fin convection to ambient	6.14	
Fin radiation + convection		5
SYSTEM THERMAL RESISTANCE		**12.1**

1. The heat pipe and the frame are closer in resistance than one would expect. This is due to the frame having about 20x more area than the heat pipe.

3 CHECK

3.1 For an acceptable design the calculated system thermal resistance must be less than the required system resistance.

3.2 The required system resistance (from #2 above): 10 °C/W

3.3 The calculated system resistance: 12.1 °C/W which is to high…

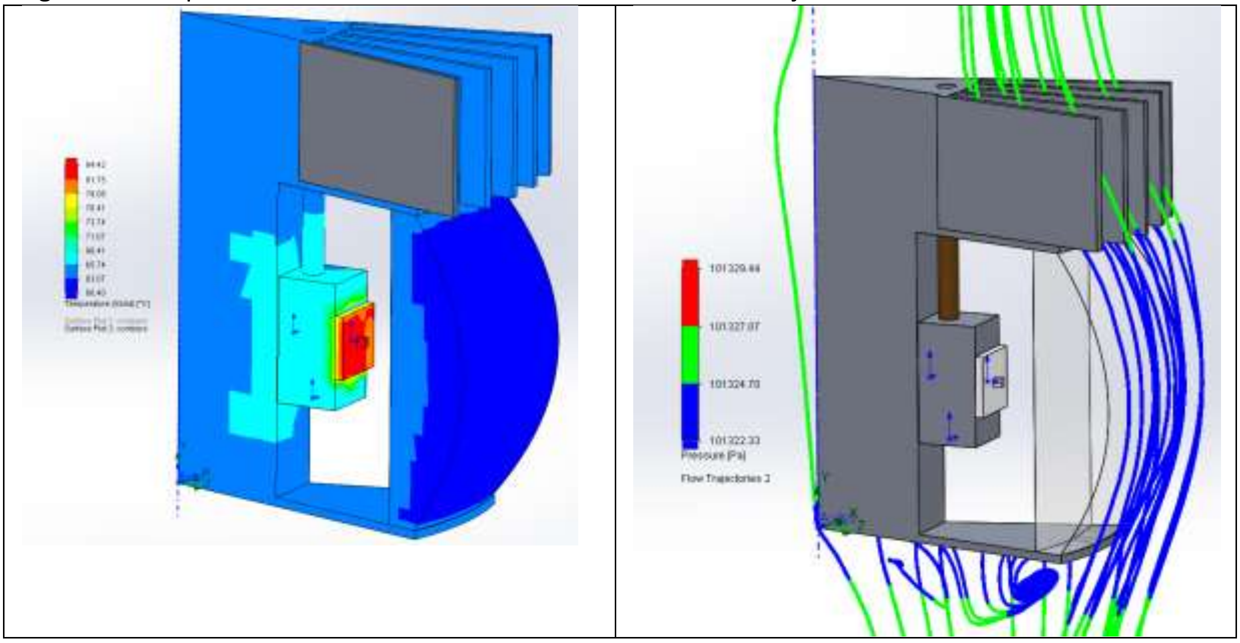
3.4 Looking at the RESISTANCE TABULATION you can see the high resistance at the Fin and the PCB. Engineering effort should be made to reduce the Fin and PCB thermal resistance.

4 REFINE OPTIMIZE with CFD ANALYSIS

4.1 Because LEDs are discrete heat sources attached to larger heat sinks or cold plates, thermal spreading resistance plays a major role in effective cooling. Thermal spreading resistance is difficult to calculate using the resistance method above so CFD software must be used to refine the analysis. Depending on design and material selection, spreading resistance is often the major impediment to the desired flow of heat. Regretfully, this is often overlooked and the result is the installation of more complex cooling systems than needed. Understanding of flow of heat from the source to the ambient, and minimizing thermal spreading resistance can be the difference between a simple thermal design and a more elaborate cooling system.

4.2 The calculations above are intended to give the designer insight into what the thermal paths are and what elements should be addressed in order to improve the thermal design. CFD software must be used to improve the thermal picture. Refine and optimize the design by using CFD simulations of the equipment.

Figure 3 Example CFD Results: Surface Plot Flow Trajectories



REFERENCES

1. Ellison Gordon "Thermal Computations for Electronics Equipment"
2. Holman J.P. "Heat Transfer"
3. DOE FUNDAMENTALS HANDBOOK THERMODYNAMICS, HEAT TRANSFER, AND FLUID FLOW Volume 2 of 3
4. http://hortsci.ashspublications.org/content/43/7/1947.full
5. http://www.slideshare.net/Audreys.Dad/fr4-pcbs-for-led-applications-testing-performance-of-pth-and-copper-pour-instead-of-mcpcbs
6. http://www.electronics-cooling.com/2006/02/a-simple-thermal-resistance-model-isoflux-versus-isothermal/